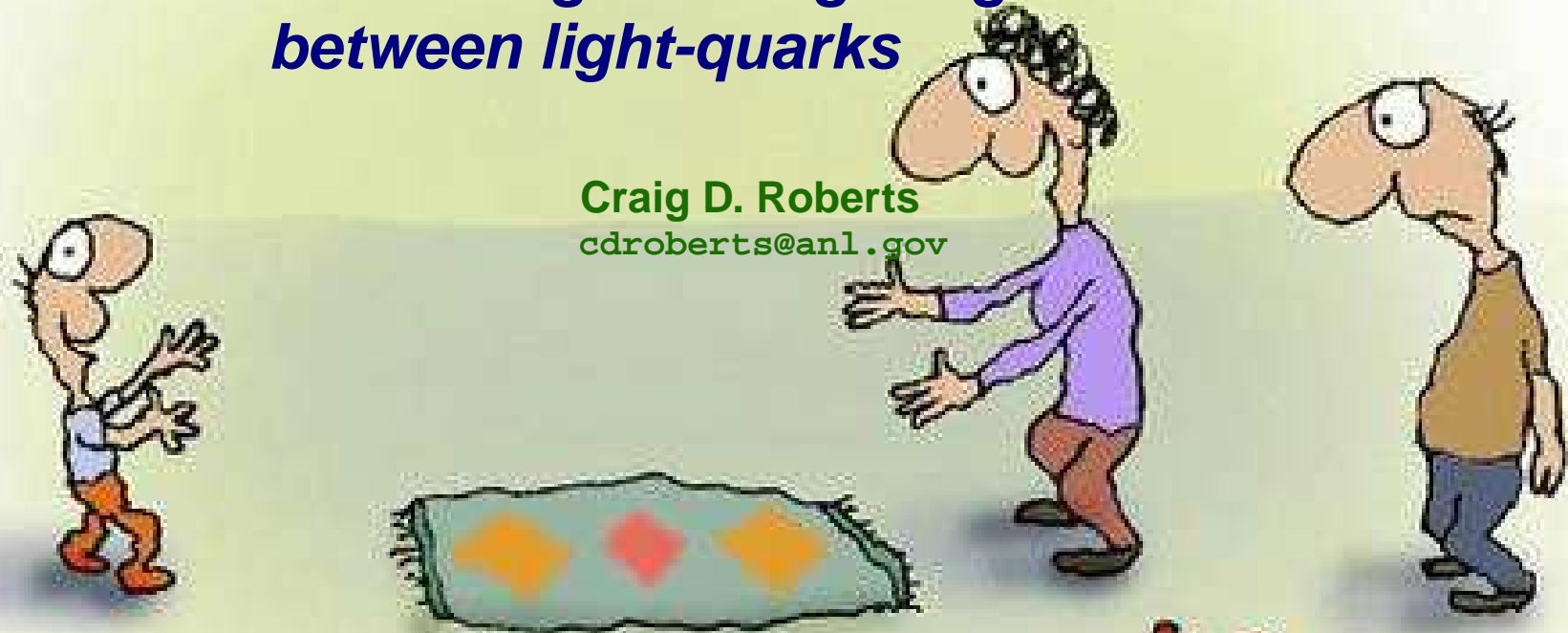


How wonderful. He's moving
with the times. He just took
his first step backwards...

Sketching the long-range interaction between light-quarks

Craig D. Roberts
cdroberts@anl.gov



Physics Division

Argonne National Laboratory

&

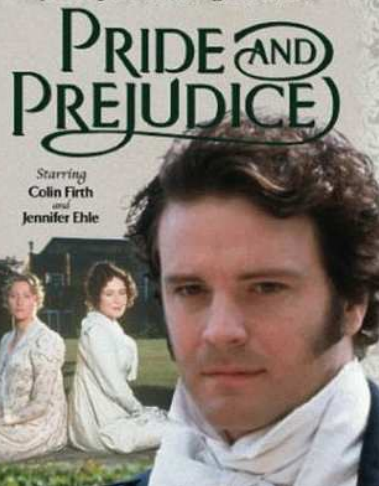
School of Physics

Peking University

<http://www.phy.anl.gov/theory/staff/cdr.html>

Universal Truths

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Universal Truths

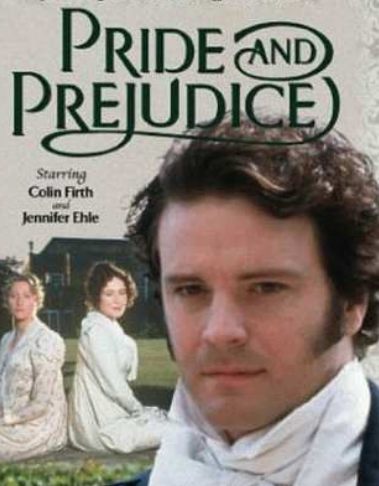


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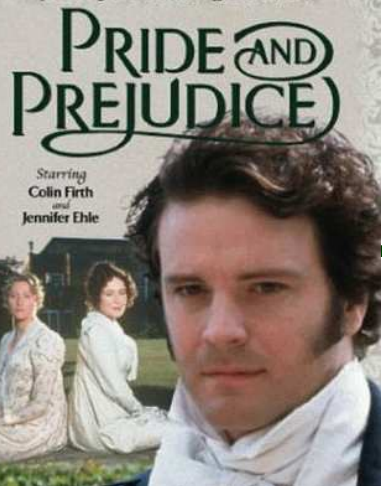
[Conclusion](#)



Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.

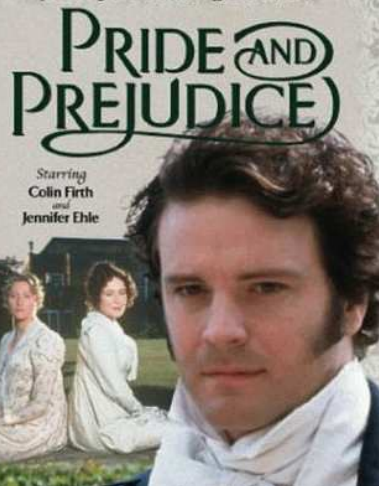




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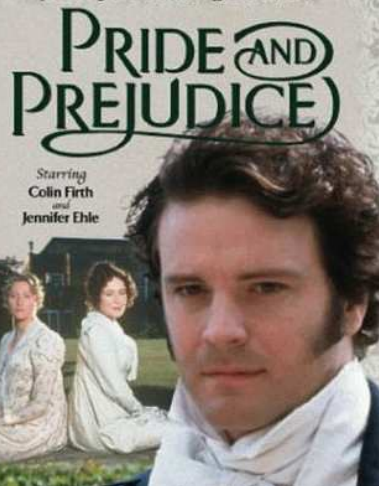




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- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. **Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.**

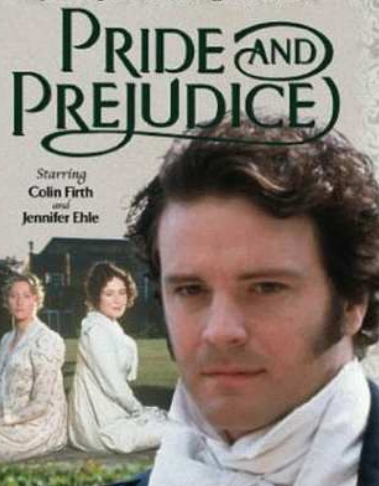




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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. **Higgs mechanism is irrelevant to light-quarks.**
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. **Problem** because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.



Dichotomy of Pion

– Goldstone Mode and Bound state





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- How does one make an **almost massless** particle
..... from two **massive** constituent-quarks?





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Current Algebra ... 1968





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The **correct understanding** of pion observables; e.g. **mass**, **decay constant** and **form factors**, **requires** an approach to contain a

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- and an **accurate realisation** of dynamical chiral symmetry breaking.





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Highly Nontrivial



What's the Problem?

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- Minimal requirements
 - detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
 - and systematic, symmetry preserving means of realising this connection in bound-states.



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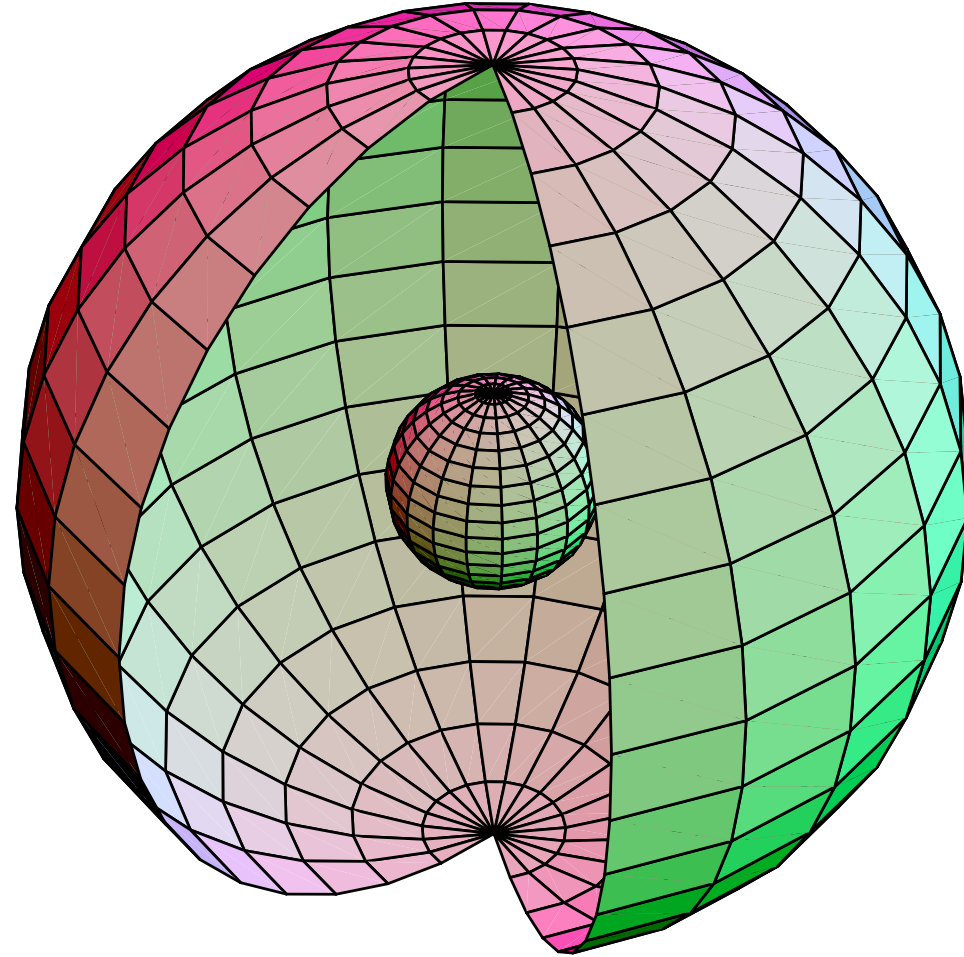
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 - Interaction between quarks – the **Interquark “Potential”** – *unknown* throughout **> 98%** of a hadron's volume



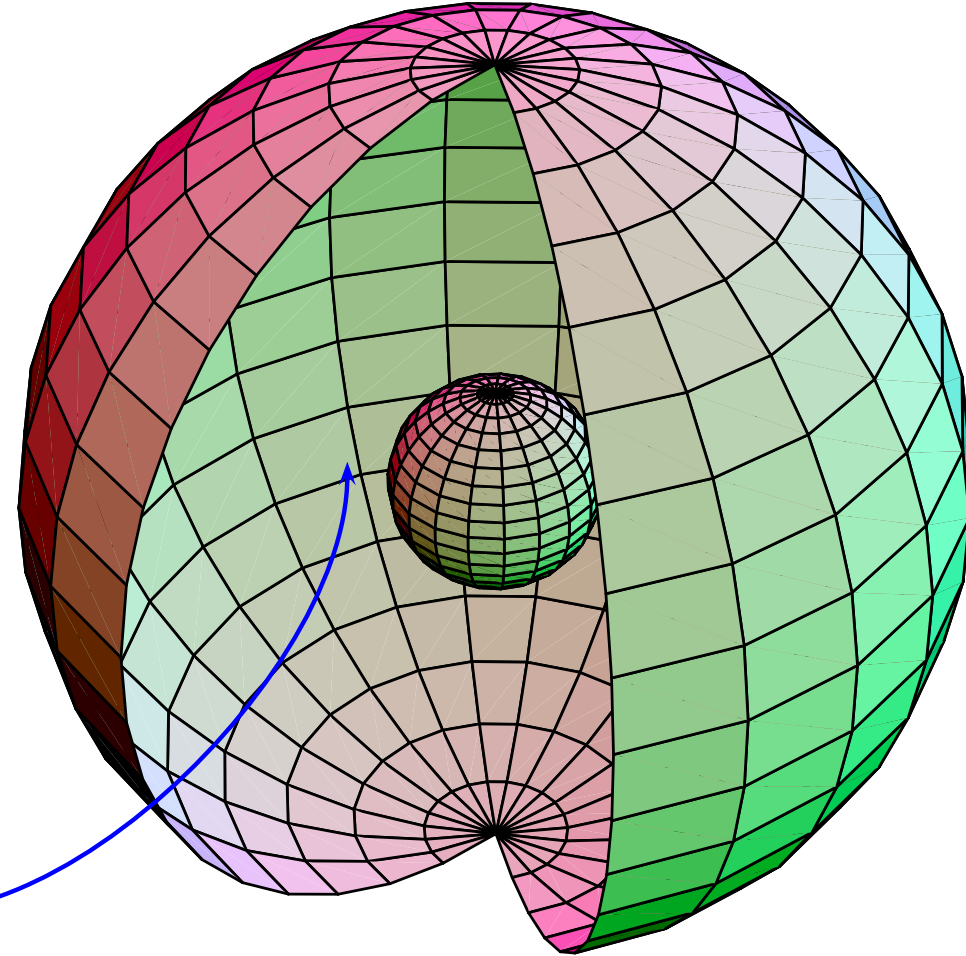
Intranucleon Interaction



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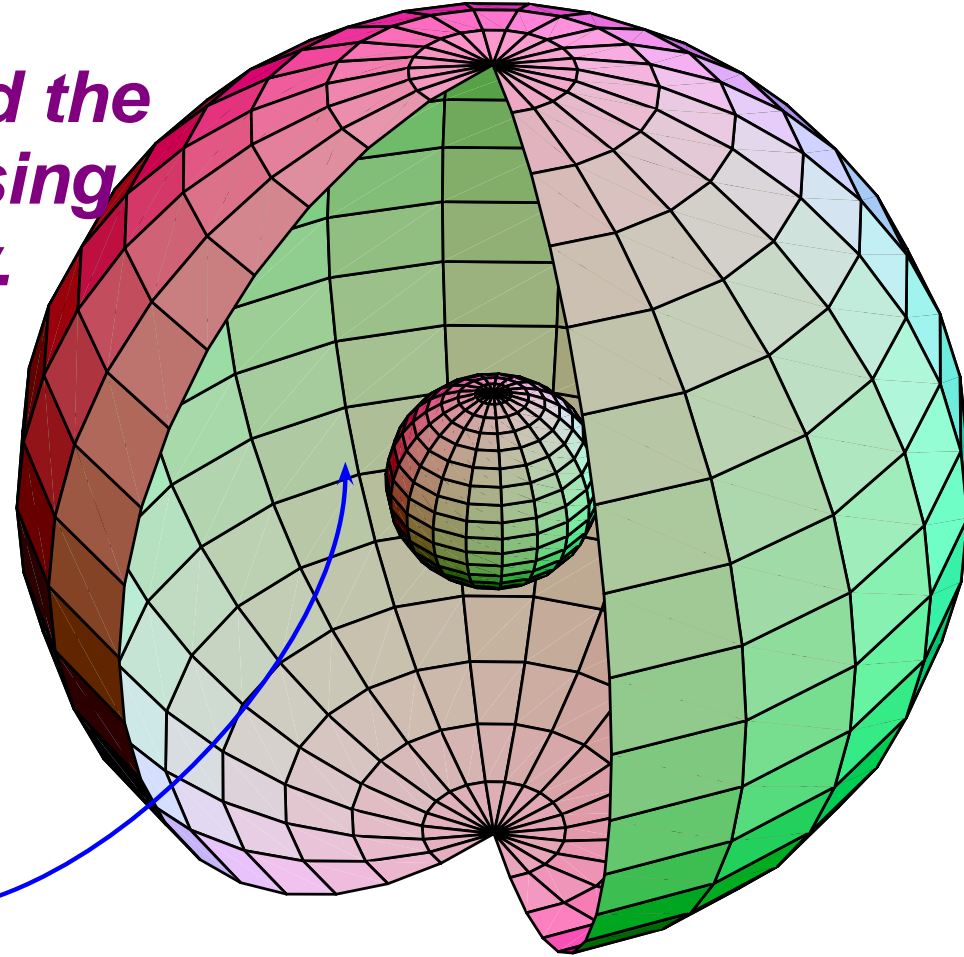


98% of the volume



What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



98% of the volume



What is the light-quark Long-Range Potential?



What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.



Charting the Interaction between light-quarks



Charting the Interaction between light-quarks

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 - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
- Of course, the behaviour of the β -function on the perturbative domain is well known.
- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.



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 - Steady quantitative progress is being made with a scheme that is systematically improvable



Charting the Interaction between light-quarks

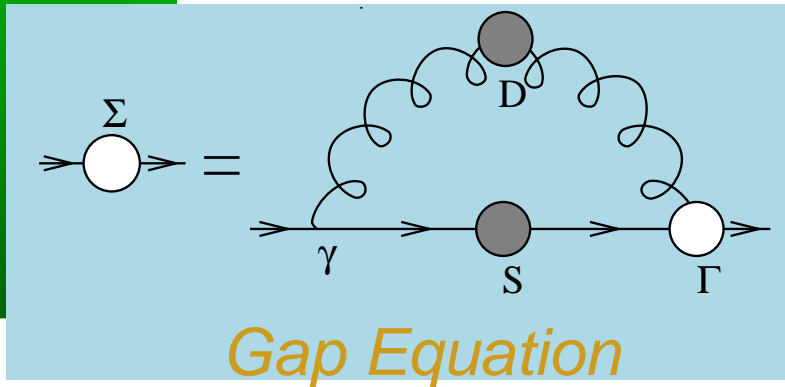
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 - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects, difficult to capture in any finite sum of contributions



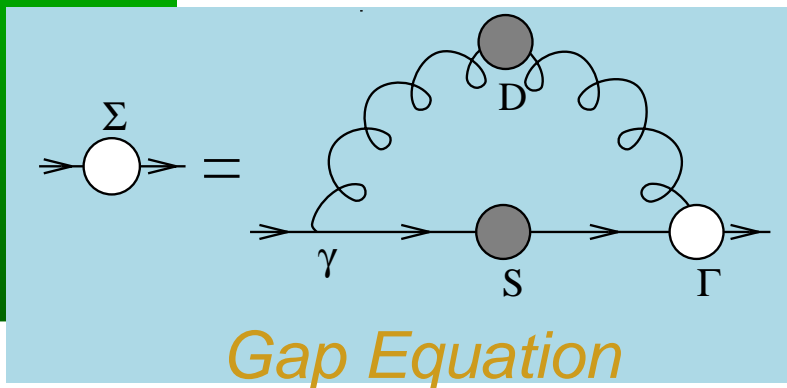
Frontiers of Nuclear Science: A Long Range Plan (2007)



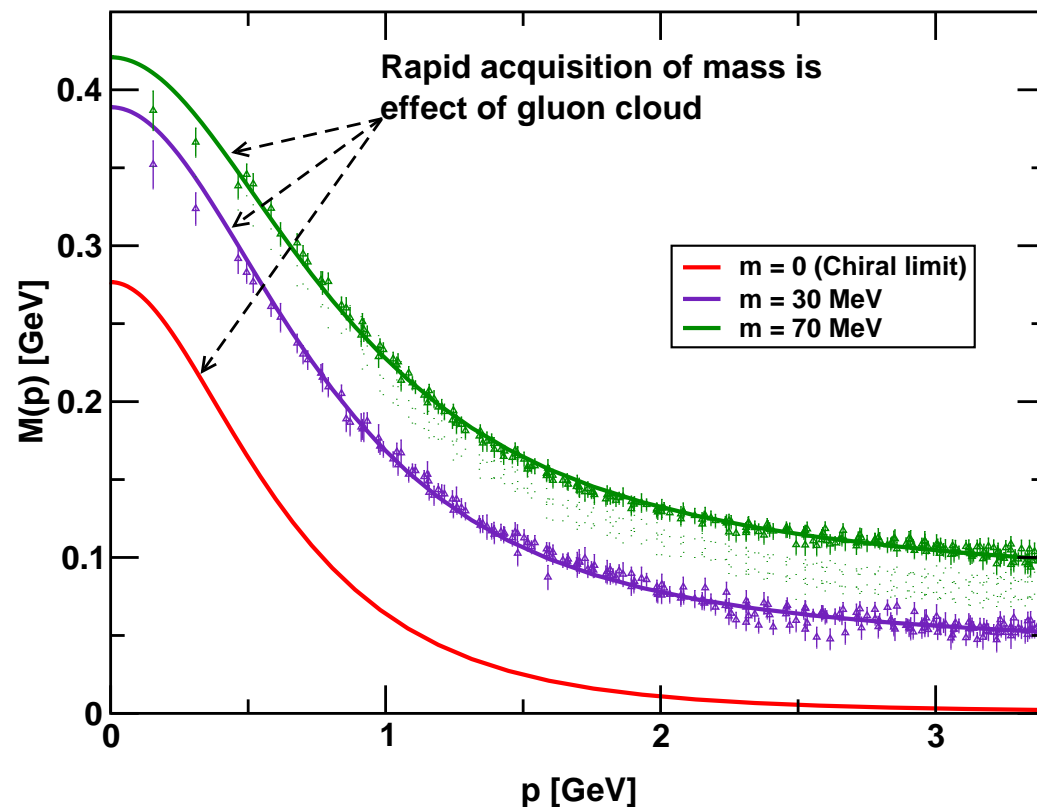
Frontiers of Nuclear Science: Theoretical Advances



Frontiers of Nuclear Science: Theoretical Advances



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

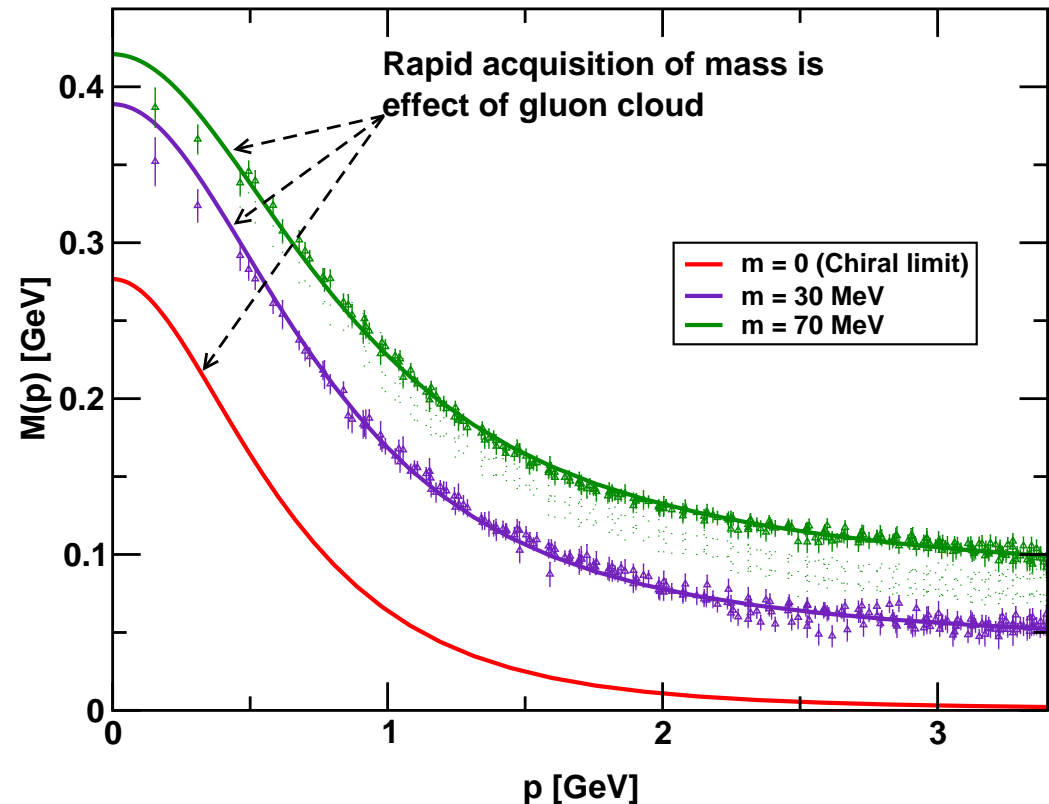


Frontiers of Nuclear Science: Theoretical Advances

Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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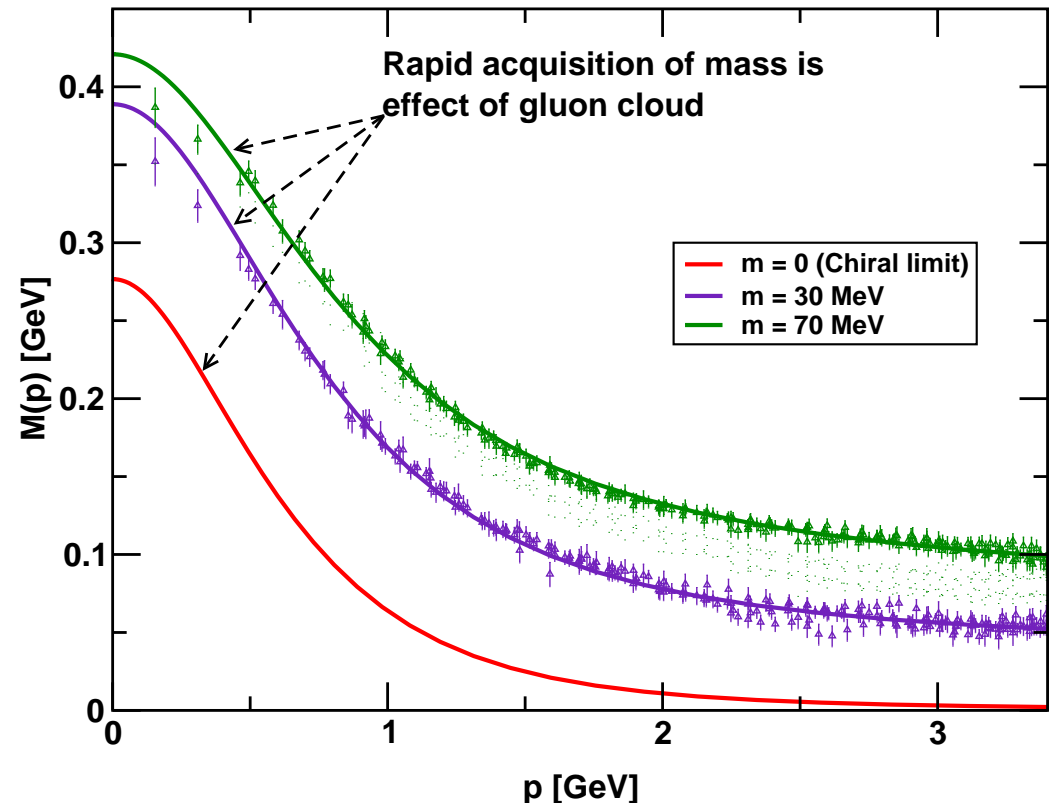


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Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



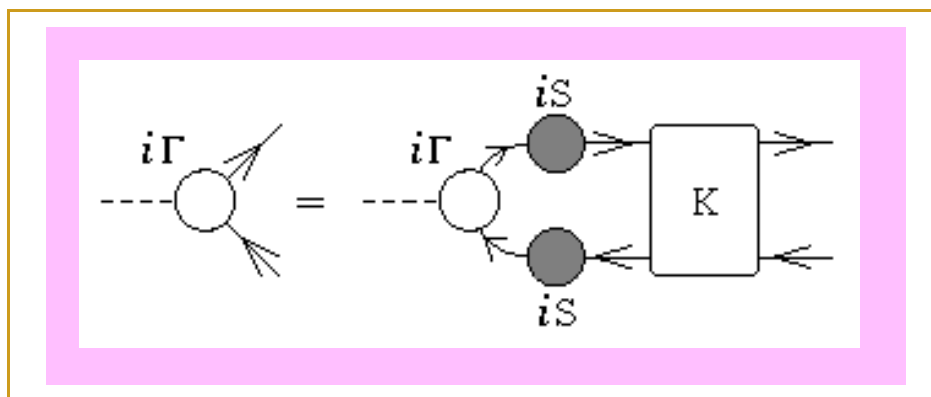
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- They appear as pole contributions to $n \geq 3$ -point colour-singlet Schwinger functions

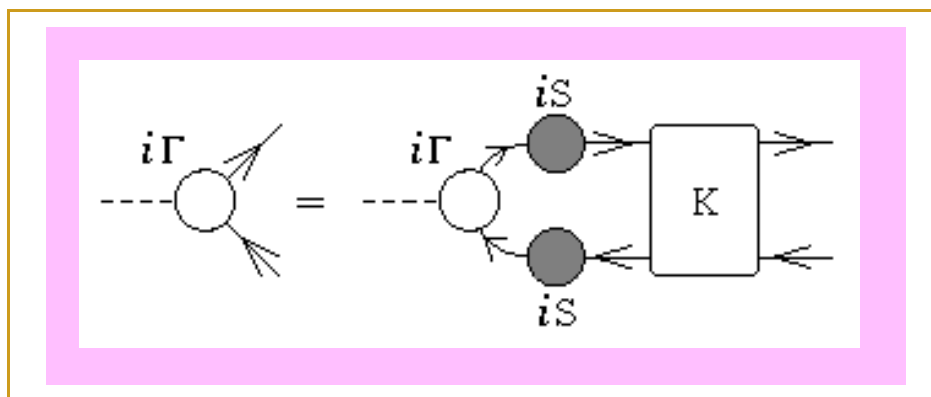


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QFT Generalisation of Lippmann-Schwinger Equation.

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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K ?
or What is the long-range potential in QCD?

Bethe-Salpeter Kernel



Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i \gamma_5 + \frac{1}{2} \lambda_f^l i \gamma_5 \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta i \Gamma_5^l(k; P) - i \Gamma_5^l(k; P) M_\zeta$$

QFT Statement of Chiral Symmetry



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Kernels very different

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- Relation **must** be preserved by truncation
- **Failure** \Rightarrow Explicit Violation of QCD's Chiral Symmetry



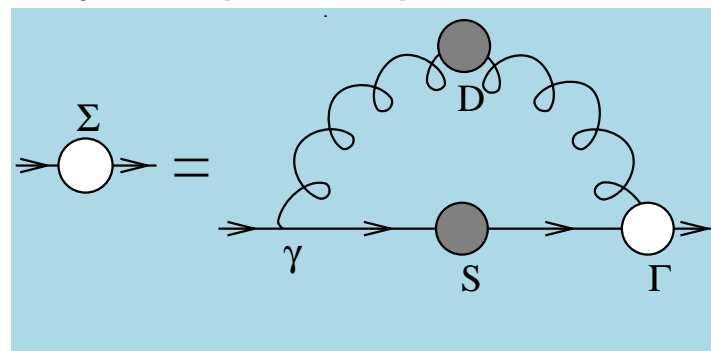
Persistent Challenge

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Persistent Challenge

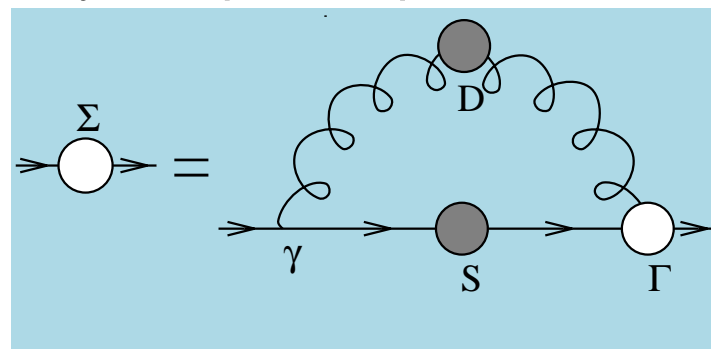
● Infinitely Many Coupled Equations





Persistent Challenge

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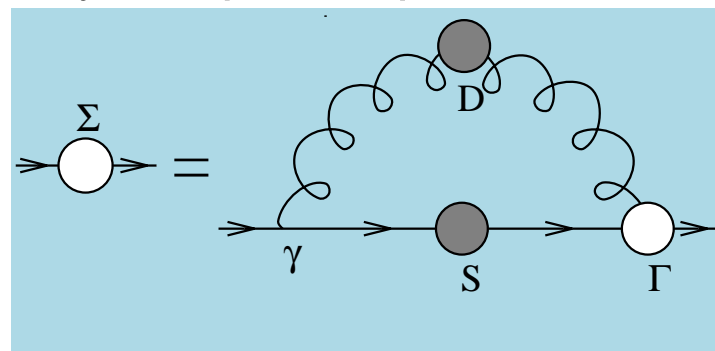
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Persistent Challenge

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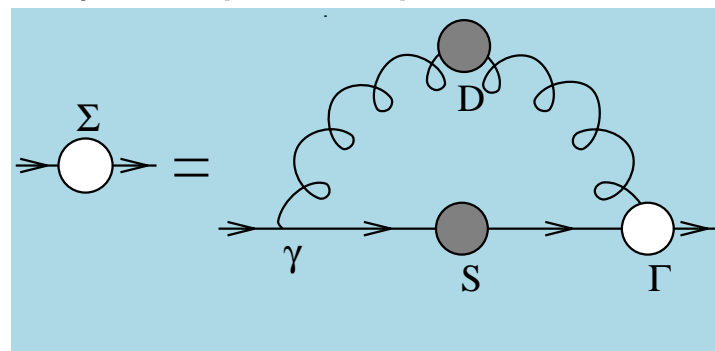
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Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation
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Not useful for the nonperturbative problems
 in which we're interested





Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme

H.J. Munczek Phys. Rev. D **52** (1995) 4736

Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation





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Radial Excitations & Chiral Symmetry

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Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$f_H \, m_H^2 = - \, \rho_\zeta^H \, \mathcal{M}_H$$



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- Mass² of pseudoscalar hadron



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
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$$f_H \ m_H^2 = - \ \rho_{\zeta}^H \ \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[M_{(\mu)} \left\{ T^H, (T^H)^{\dagger} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



Radial Excitations & Chiral Symmetry

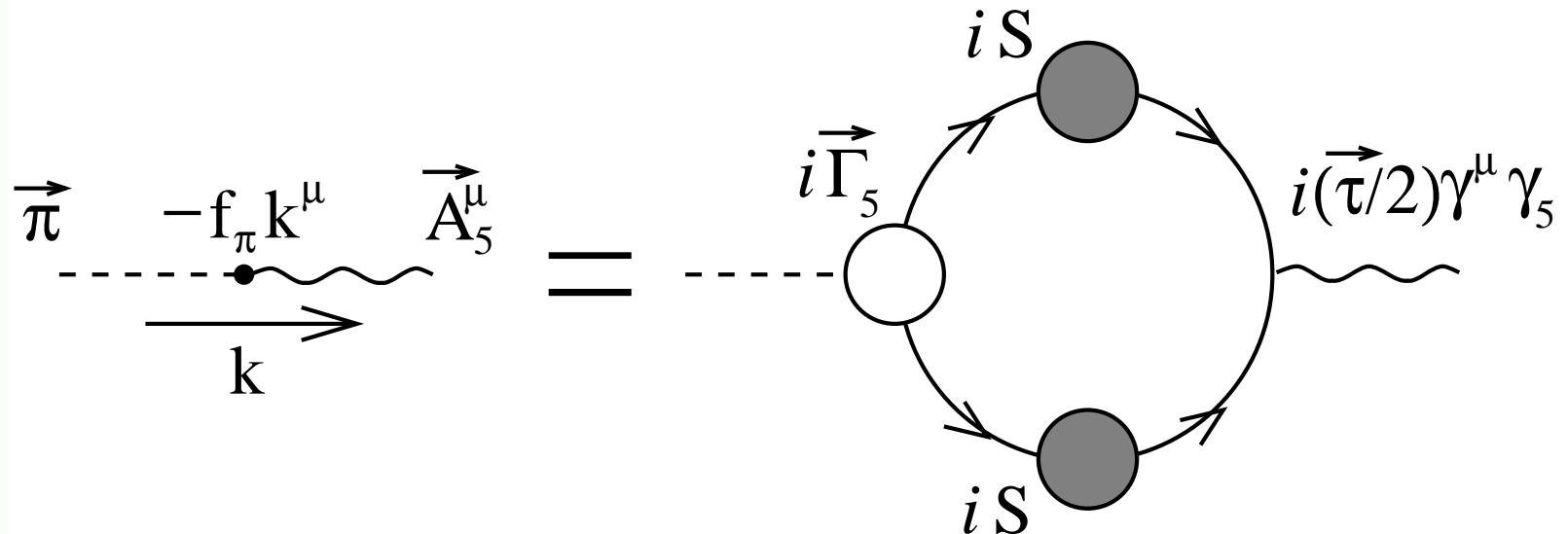
(Maris, Roberts, Tandy
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$$\langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle$$

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$f_H p_\mu = Z_2 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

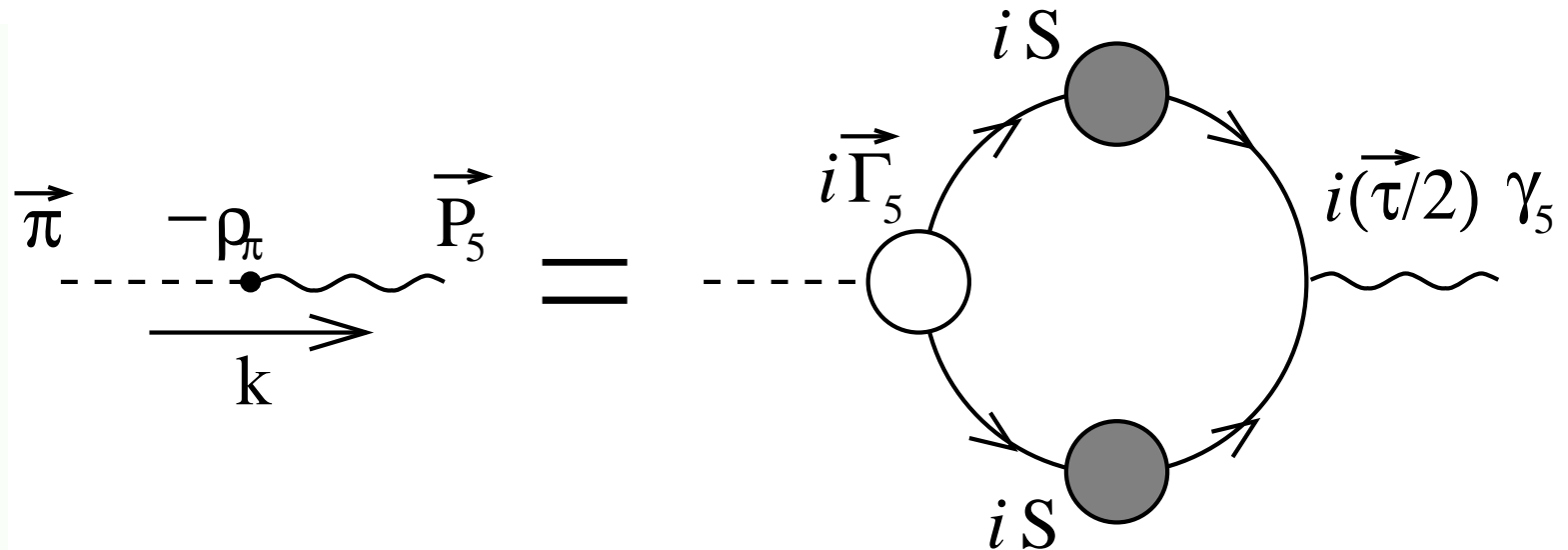
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● Light-quarks; i.e., $m_q \sim 0$

● $f_H \rightarrow f_H^0$ & $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$ GMOR relation, a corollary



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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

Craig Roberts: *Sketching the long-range interaction between light-quarks*

GHP 2009, 29/04-01/05, Denver Co ... 27

- p. 14/35



Gap Equation

General Form

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- Return to general bound-state problem . . .



- Return to general bound-state problem ...
- To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$
$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q, p),$$



- Return to general bound-state problem ...
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- $D_{\mu\nu}(k)$ is the dressed-gluon propagator;
- $\Gamma_\nu^f(q, p)$ is the dressed-quark-gluon vertex;
- $m^{\text{bm}}(\Lambda)$ is the Lagrangian current-quark bare mass;
- $Z_{1,2}(\zeta^2, \Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point.



Bethe-Salpeter Equation

General Form



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- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.



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$$\begin{aligned}\Gamma_{5\mu}^{fg}(k; P) = & Z_2 \gamma_5 \gamma_\mu - \int_q g^2 D_{\alpha\beta}(k - q) \\ & \times \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-) \\ & + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),\end{aligned}$$



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- $\Lambda_{5\mu\beta}^{fg}$ is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g., $q_\pm = q \pm P/2$, etc., without loss of generality



Ward-Takahashi Identity

Bethe-Salpeter Kernel



Ward-Takahashi Identity

Bethe-Salpeter Kernel

- In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) - i [m_f(\zeta) + m_g(\zeta)] \Gamma_5^{fg}(k; P),$$

expresses chiral symmetry & pattern by which it's broken



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NECESSARY & SUFFICIENT

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- Rainbow-ladder ...

- $$\Gamma_\beta^f(q, k) = \gamma_\mu$$

$$\Rightarrow \Lambda_{5\mu\beta}^{fg}(k, q; P) = 0 = \Lambda_{5\beta}^{fg}(k, q; P)$$



Bethe-Salpeter Kernel

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Bethe-Salpeter Kernel

60 year problem

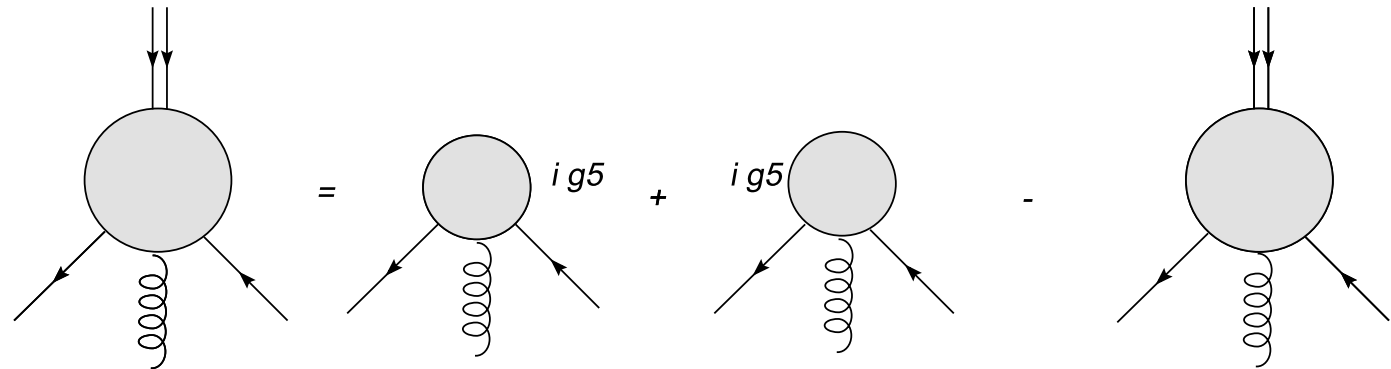
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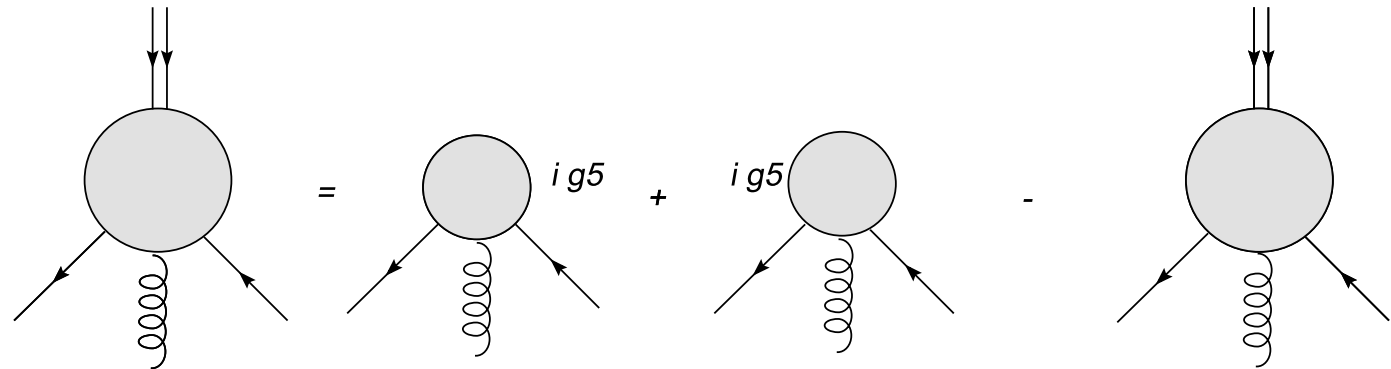


Bethe-Salpeter Kernel

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Chang Lei (IAPCM, Beijing) & CDR
arXiv:0903.5461 [nucl-th]

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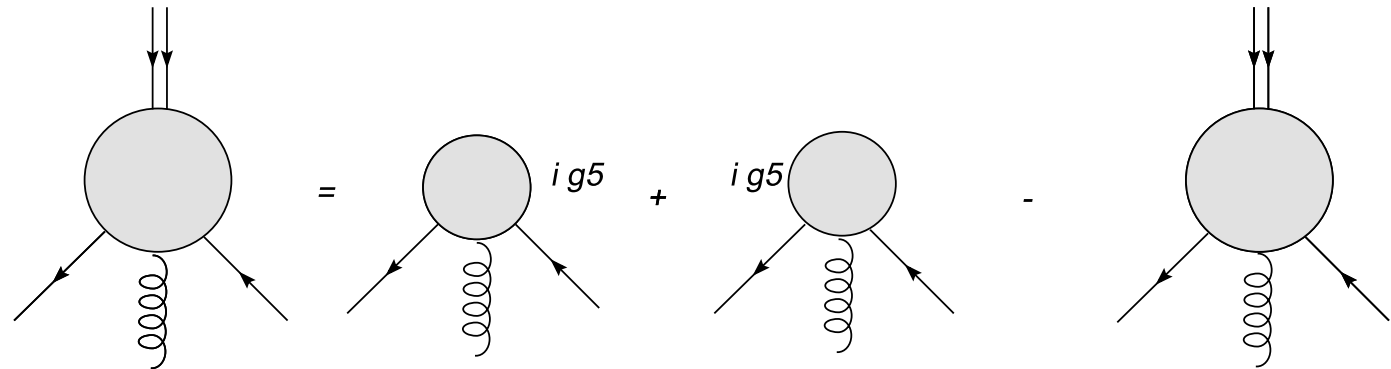
- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
 - Consistent means - all symmetries preserved!



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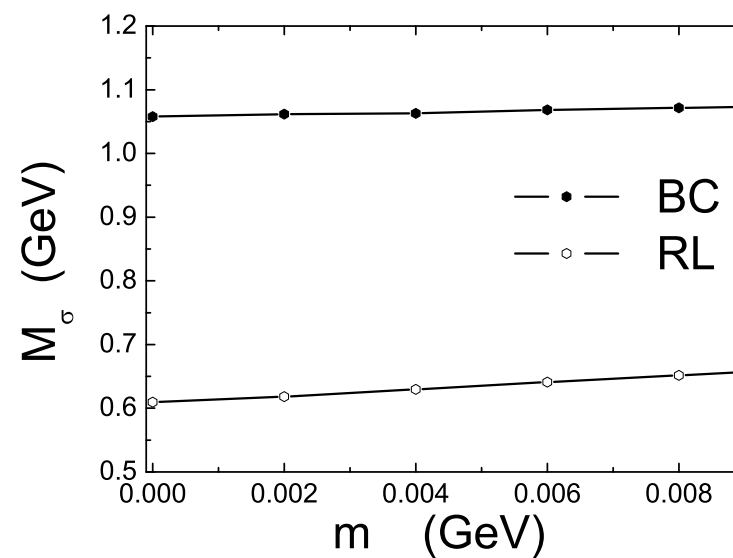
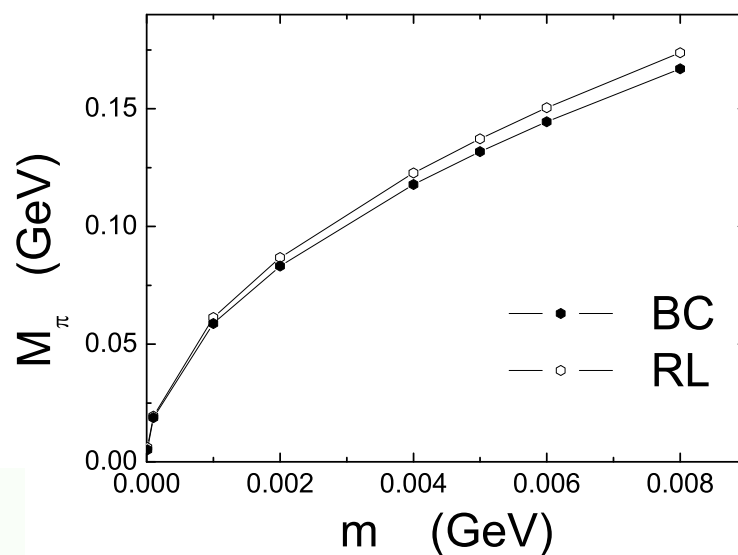
- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Exemplified the procedure and results to expect ...



Argonne
NATIONAL
LABORATORY

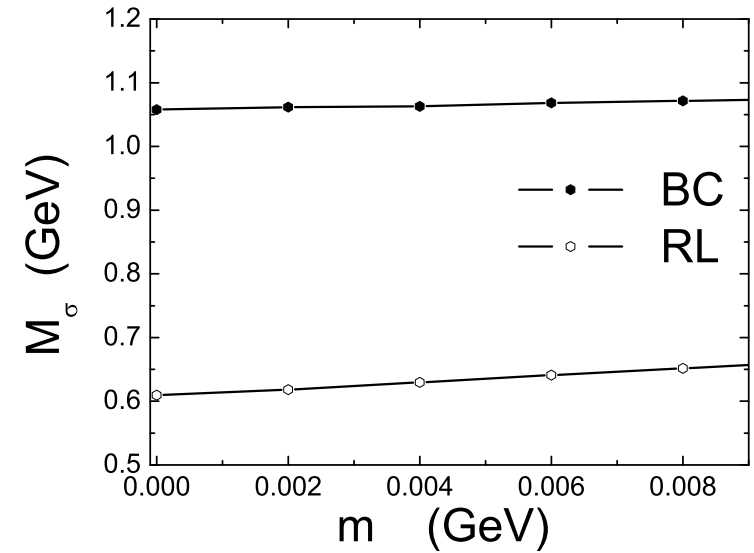
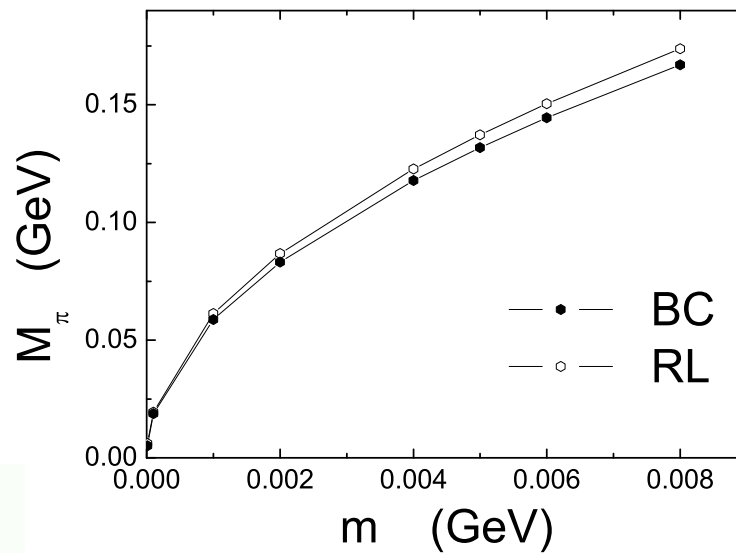
Numerical Illustration

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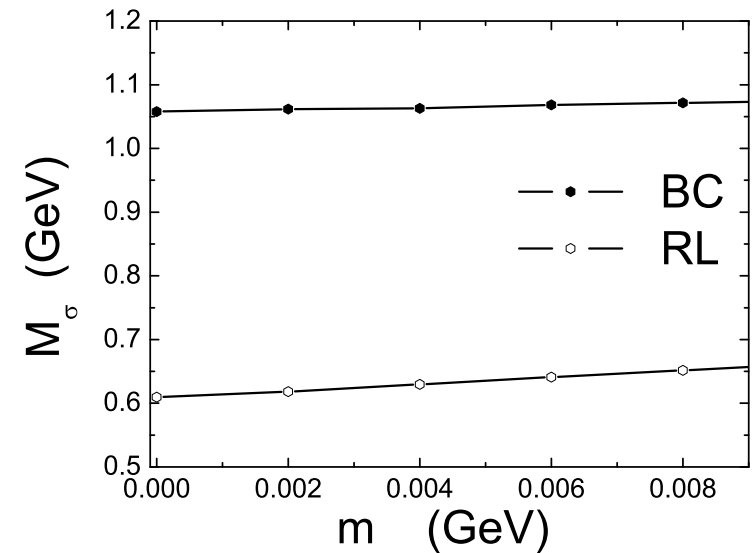
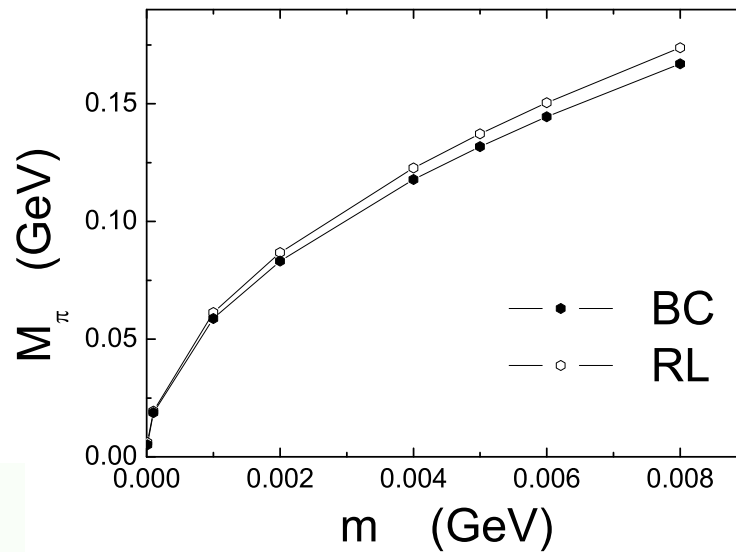
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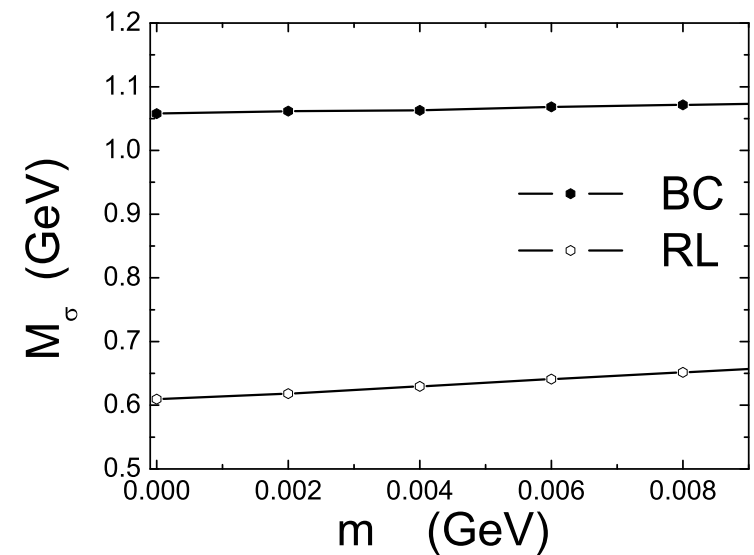
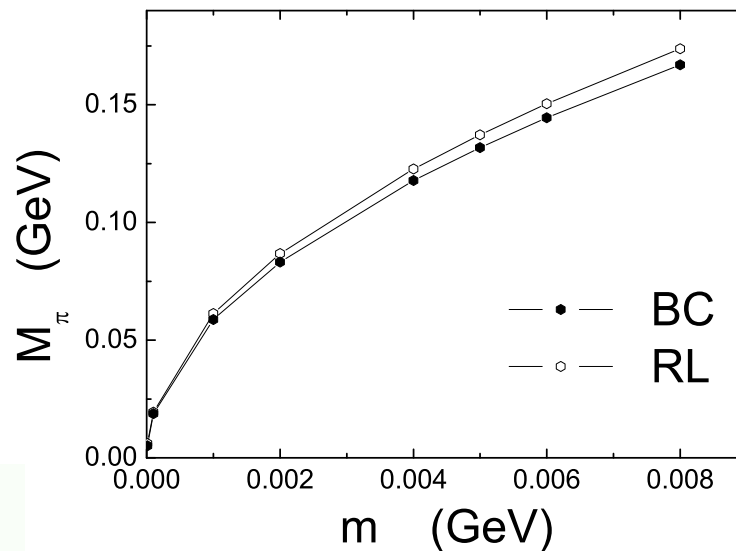
● Single interaction, common mass scale:
rainbow-ladder cf. BC-consistent truncation





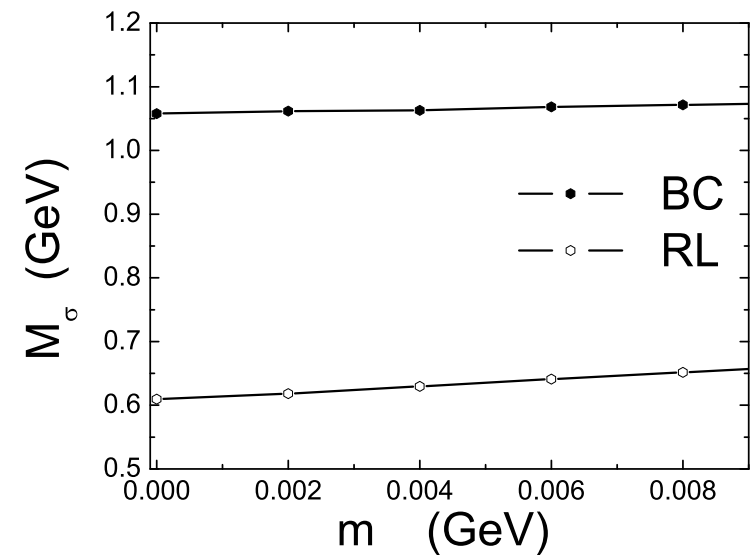
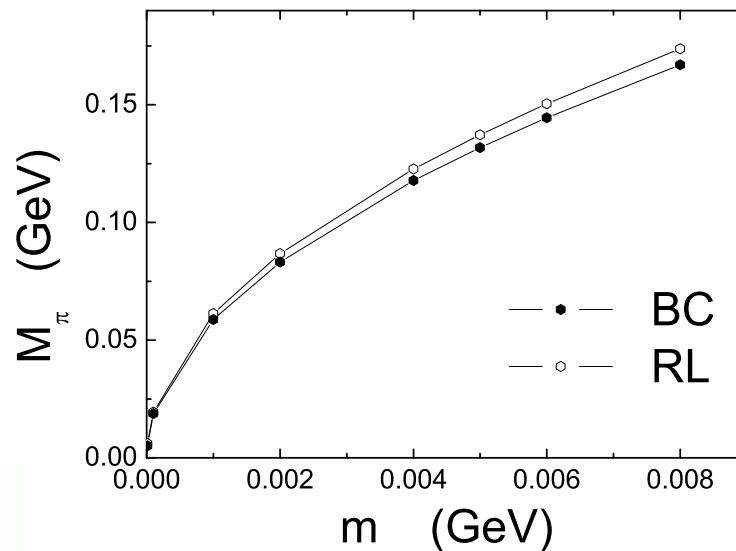
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Spin-orbit Interaction

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]

- Rainbow-ladder DSE truncation,

$$\varepsilon_{\sigma}^{\text{RL}} := \left. \frac{2M(0) - m_{\sigma}}{2M(0)} \right|_{\text{RL}} = (0.3 \pm 0.1) .$$

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- From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson



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- Scalar mesons = 3P_0 states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting



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- Scalar mesons = 3P_0 states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.
Impossible to demonstrate effect without our new procedure



Spin-orbit Interaction

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- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
prima facie ... can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small



Spin-orbit Interaction

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- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
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- Promise of **realistic** meson spectroscopy
First time, also for mass > 1 GeV





Unifying Study of Mesons and Baryons



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- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons?



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Unifying Study of Mesons and Baryons

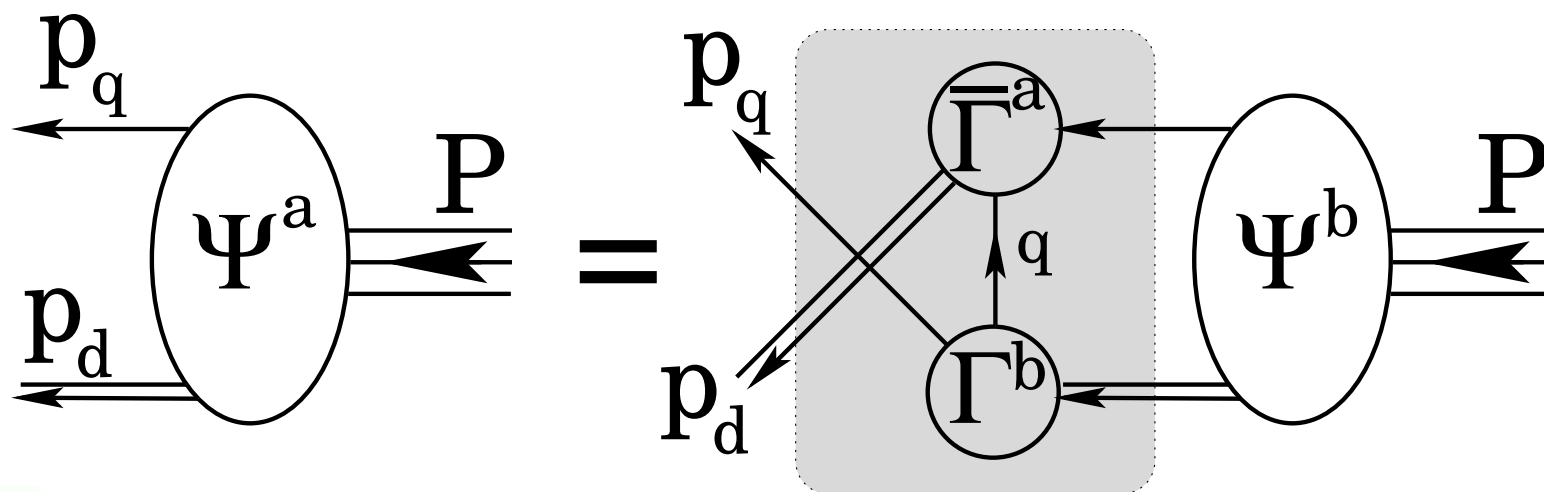
- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.
- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
 - Residue is proportional to nucleon's Faddeev amplitude
 - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
 - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour- $\bar{3}$ (antitriplet) channel



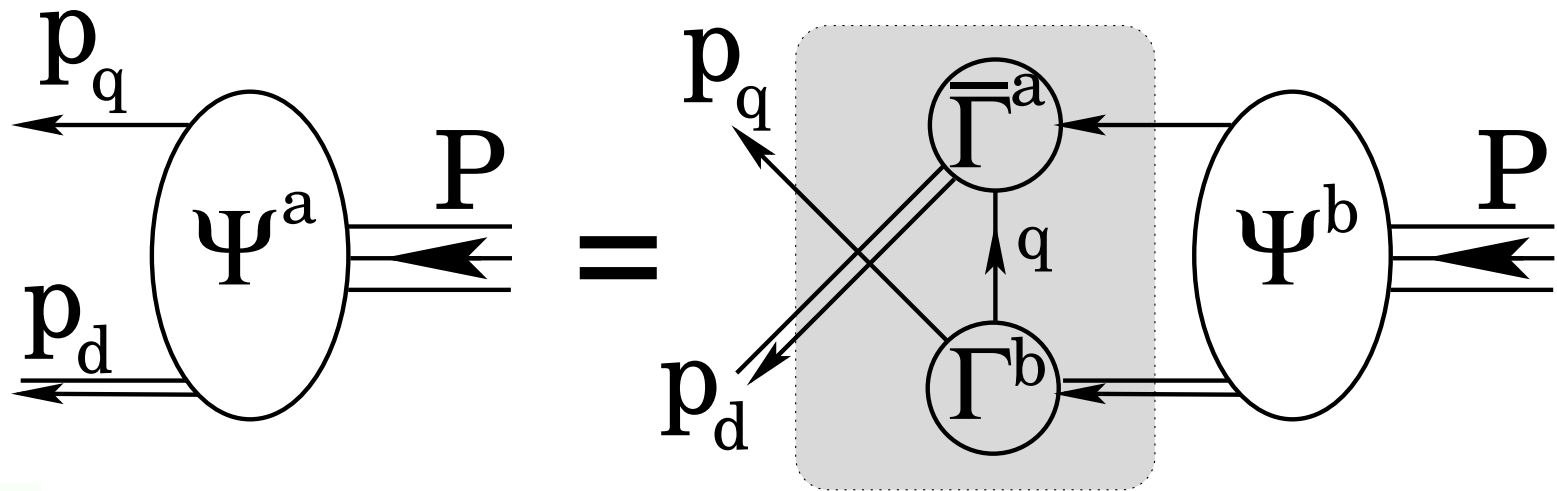
Faddeev equation



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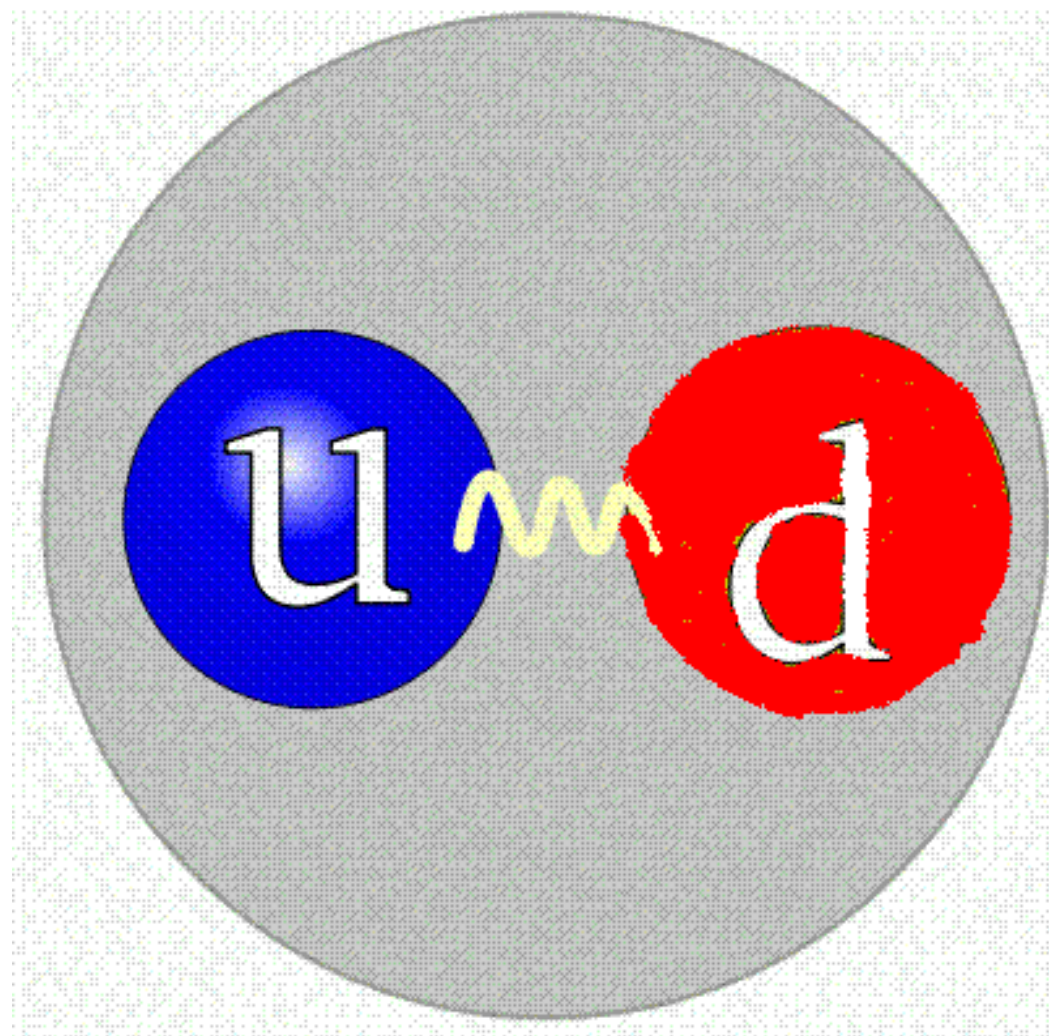
Faddeev equation



- Linear, Homogeneous Matrix equation
 - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... *s*–, *p*– & *d*–wave correlations



Diquark correlations



Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red

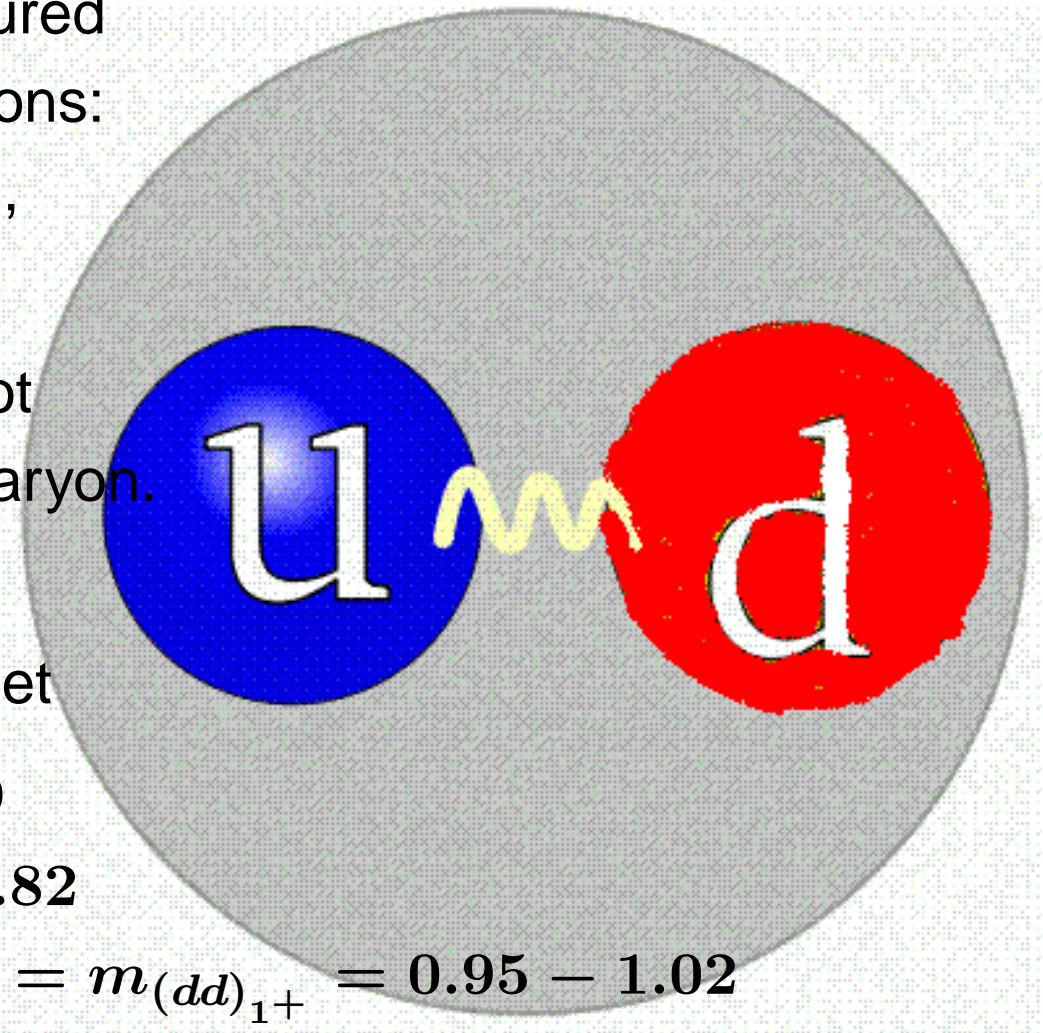
- Confined ... Does not escape from within baryon.

- Scalar is isosinglet, Axial-vector is isotriplet

- DSE and lattice-QCD

$$m_{[ud]_{0+}} = 0.74 - 0.82$$

$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$



Nucleon-Photon Vertex



M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

6 terms . . .

Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude



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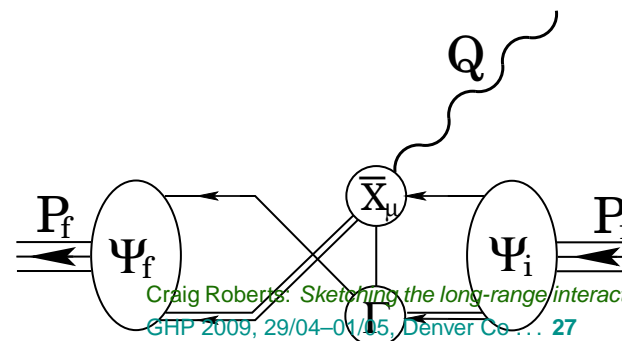
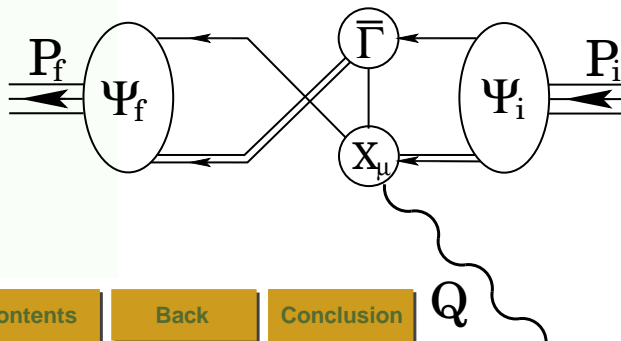
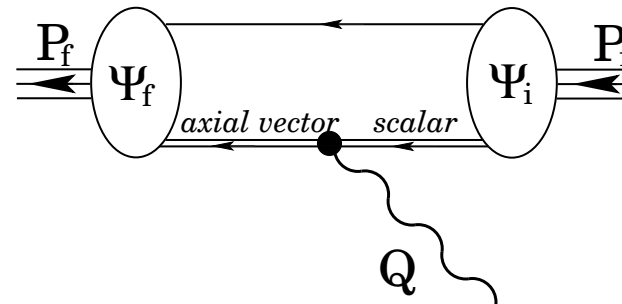
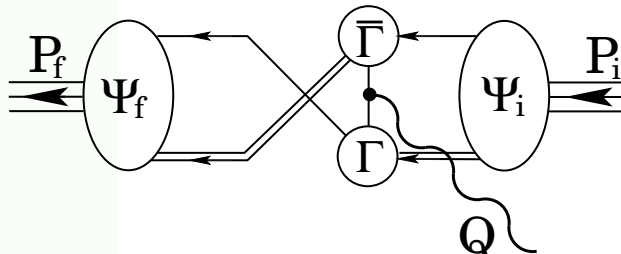
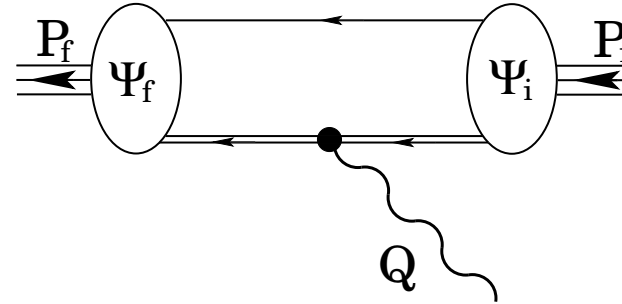
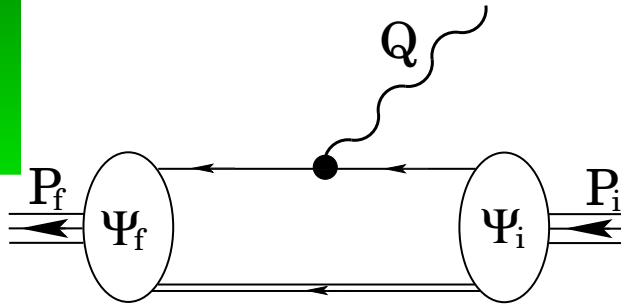
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6 terms ...

Nucleon-Photon Vertex

constructed systematically ... current conserved automatically
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$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$





Cloët *et al.*

– arXiv:0710.2059 [nucl-th]

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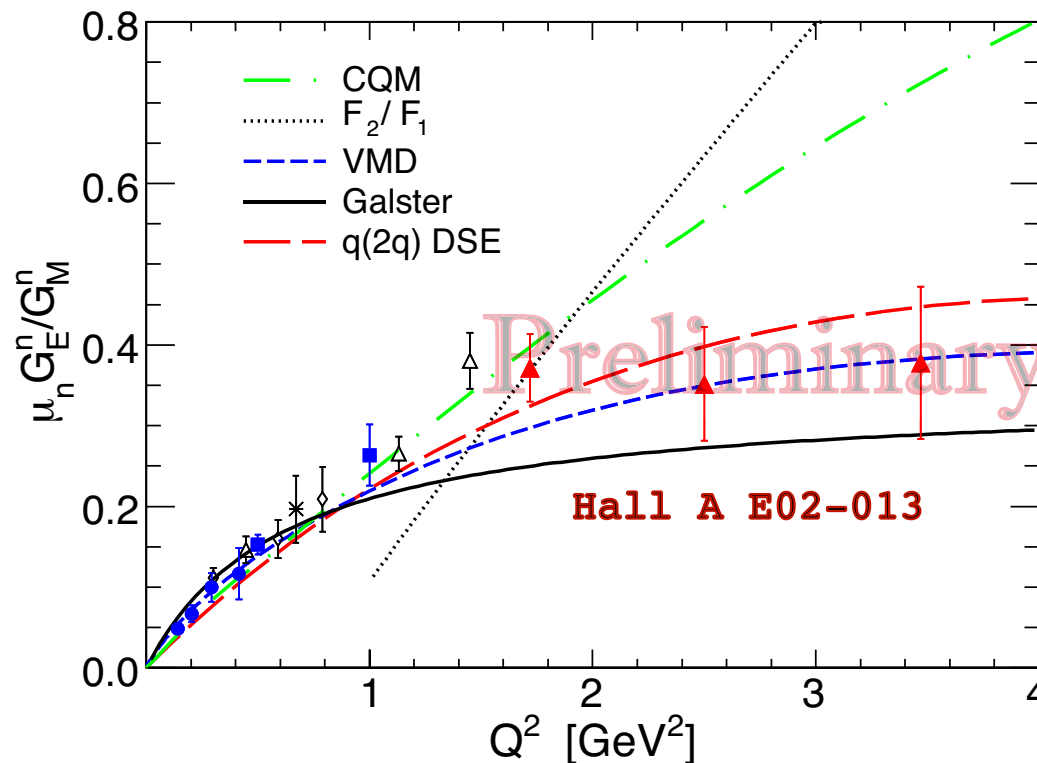
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● DSE-Faddeev Equation prediction



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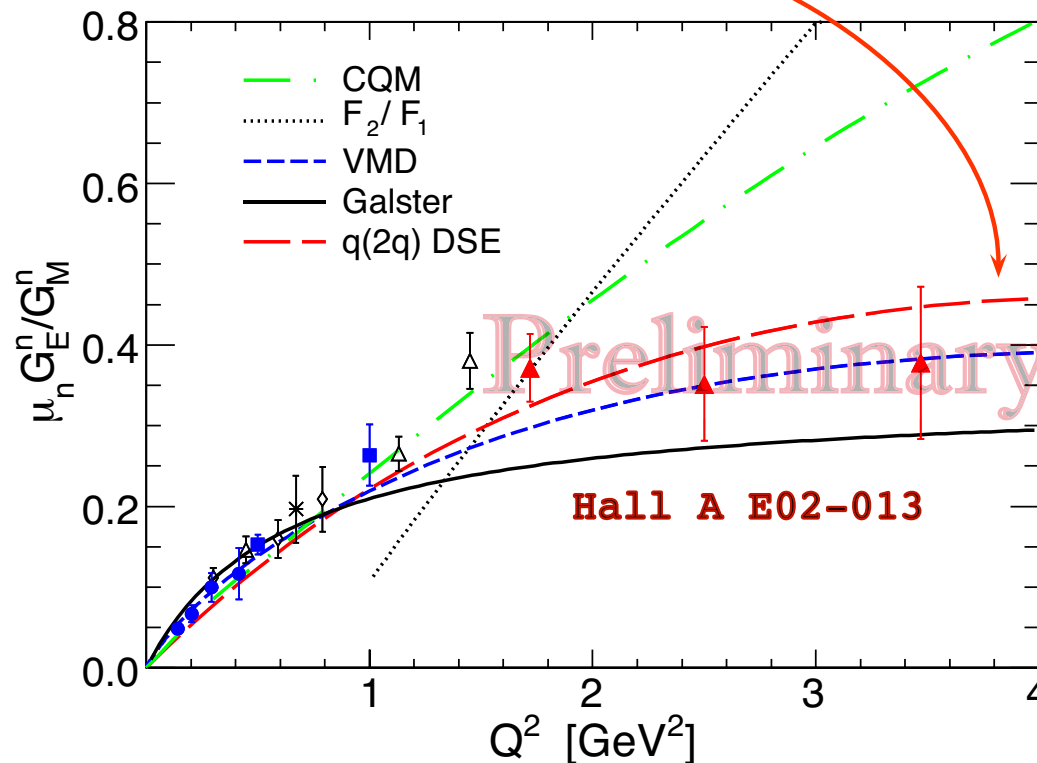
Jefferson Lab E02-013 Collaboration, *in preparation*.



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● DSE-Faddeev Equation prediction

Red long-dashed curve



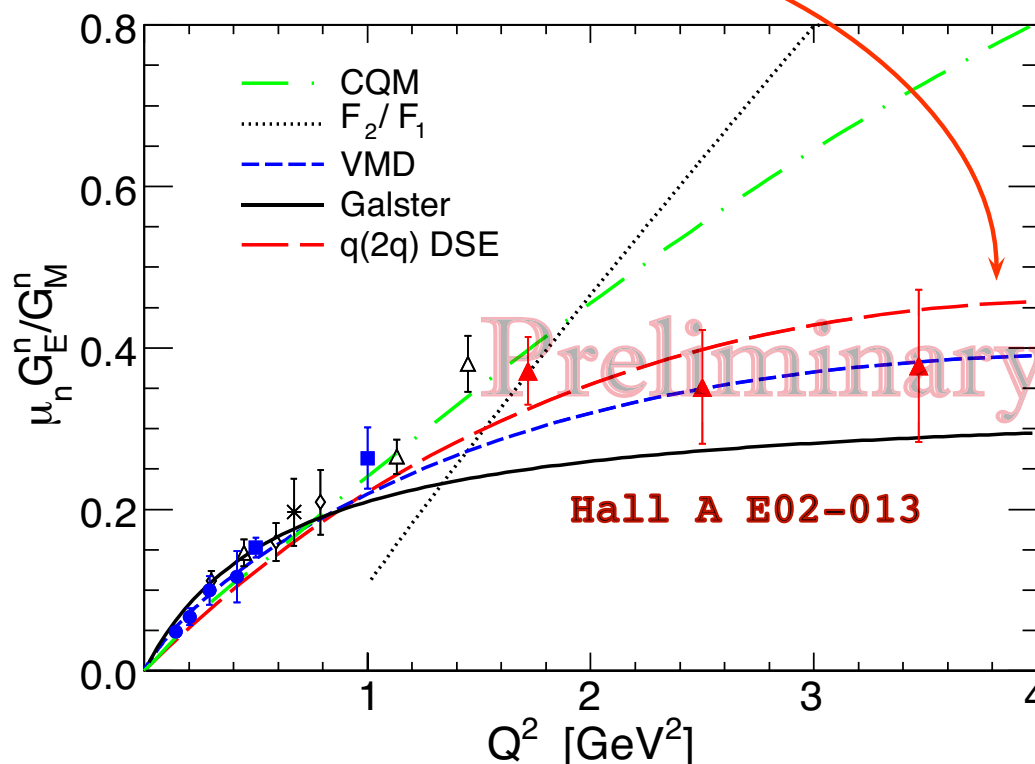
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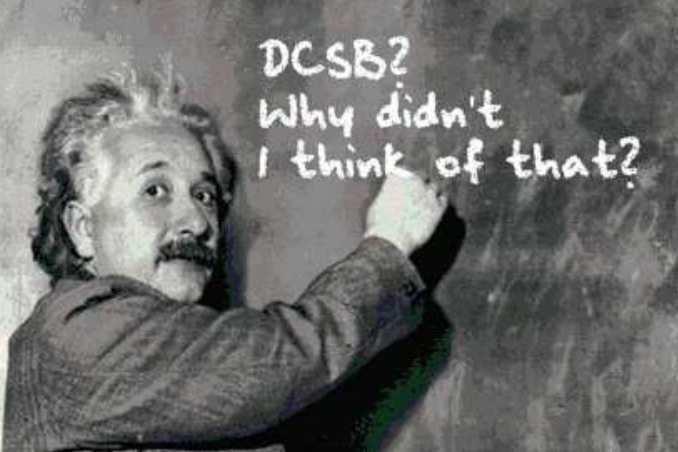
Session L4: Nucleon Microscopy, Sunday, May 3, 3:30PM:

Bogdan Wojtsekhowski



Epilogue

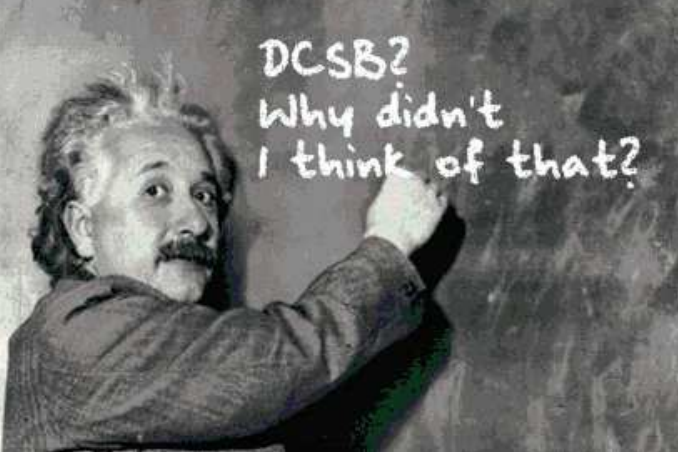




- DCSB exists in QCD.

Epilogue



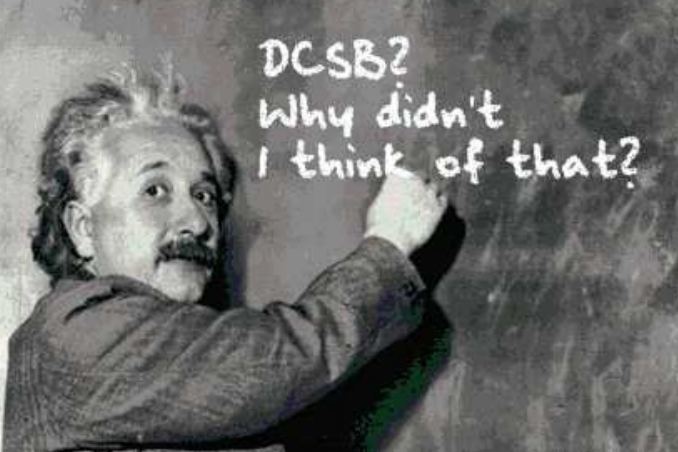


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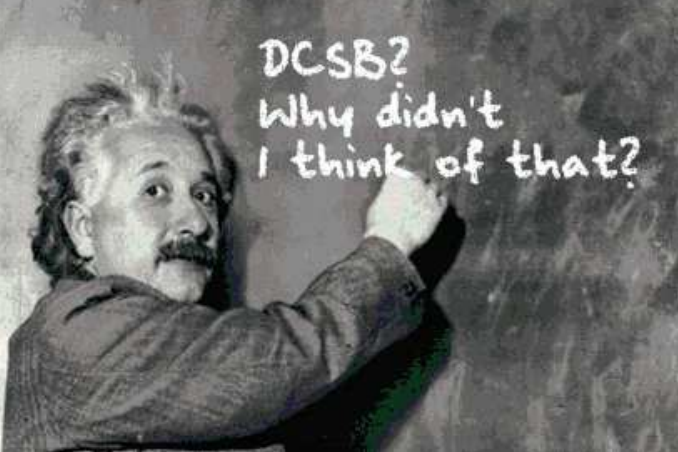


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 - light current-quarks become heavy constituent-quarks: $4 \rightarrow 400 \text{ MeV}$
 - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140 \text{ MeV}$
 - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
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 $g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$



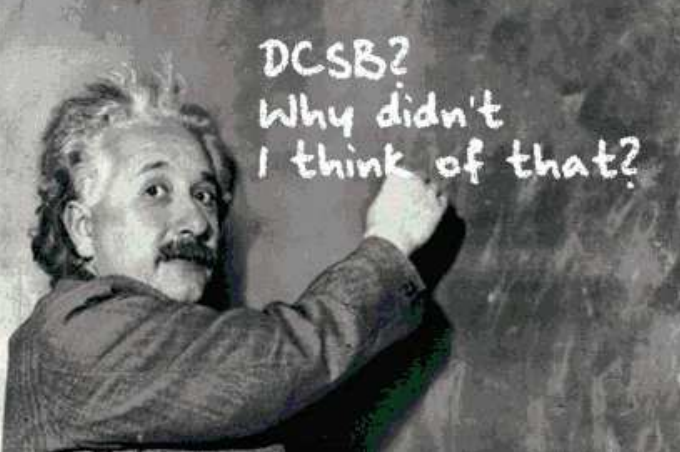


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$$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$
- It impacts dramatically upon observables.

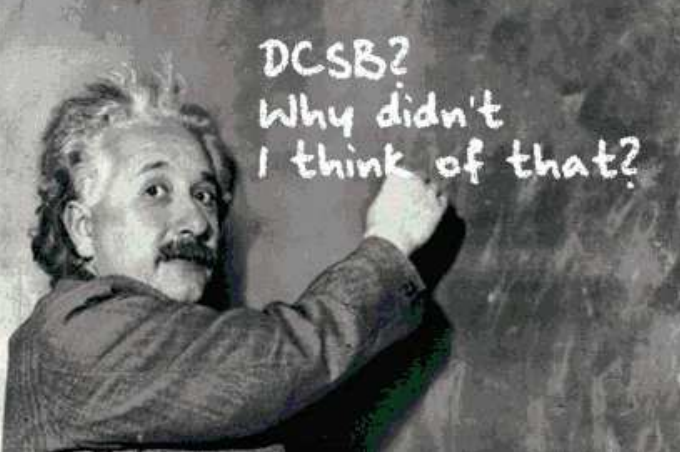




Epilogue

- Dyson-Schwinger Equations
 - Poincaré covariant unification of meson and baryon observables

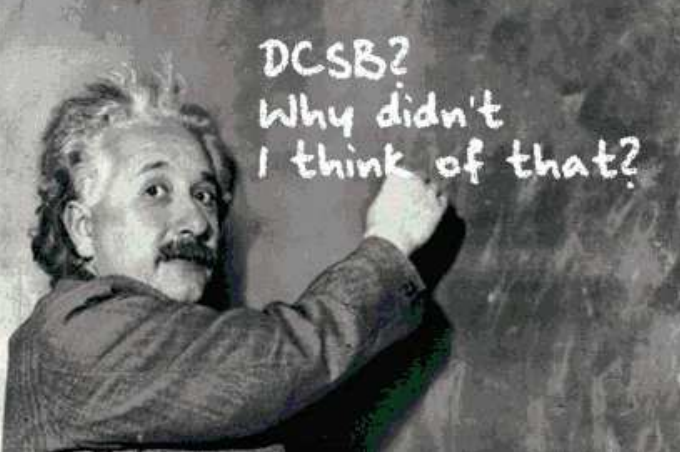




Epilogue

- Dyson-Schwinger Equations
 - Poincaré covariant unification of meson and baryon observables
 - All global and pointwise corollaries of DCSB are manifested naturally without fine-tuning

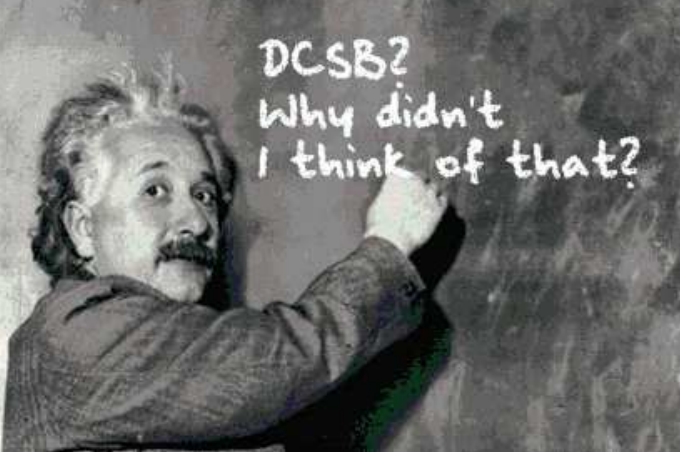




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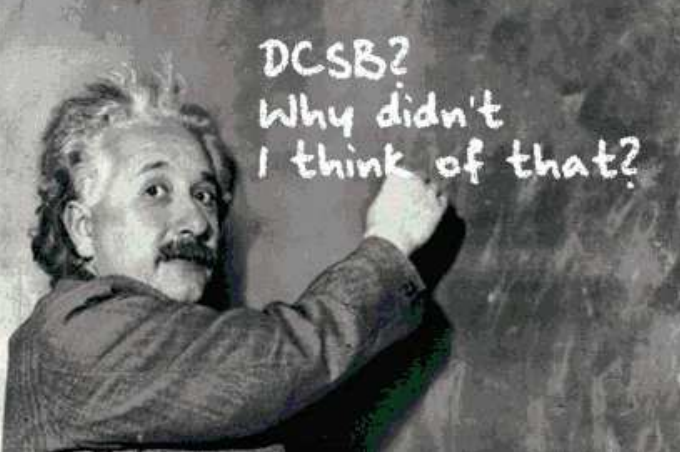




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- Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks



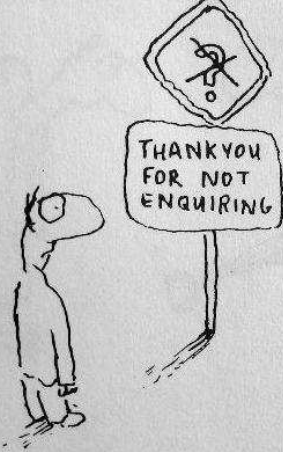
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2. QCD's Challenges
3. Dichotomy of the Pion
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7. Bethe-Salpeter Kernel
8. Persistent Challenge
9. BSE General Form
10. Unifying Meson & Nucleon
11. Faddeev equation
12. Nucleon-Photon Vertex
13.
$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$
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17. Diquark correlations
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19. Ratio of Neutron
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20. Pion Cloud



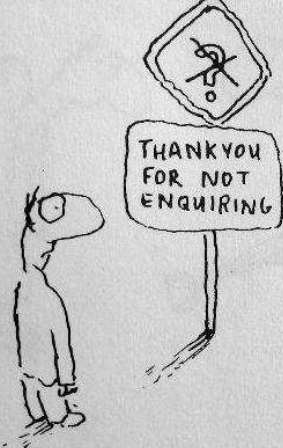
QCD's Challenges





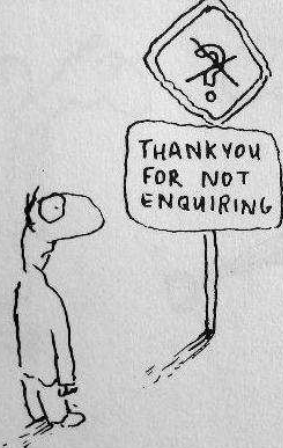
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Understand Emergent Phenomena

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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour
arises from apparently simple rules



Confinement



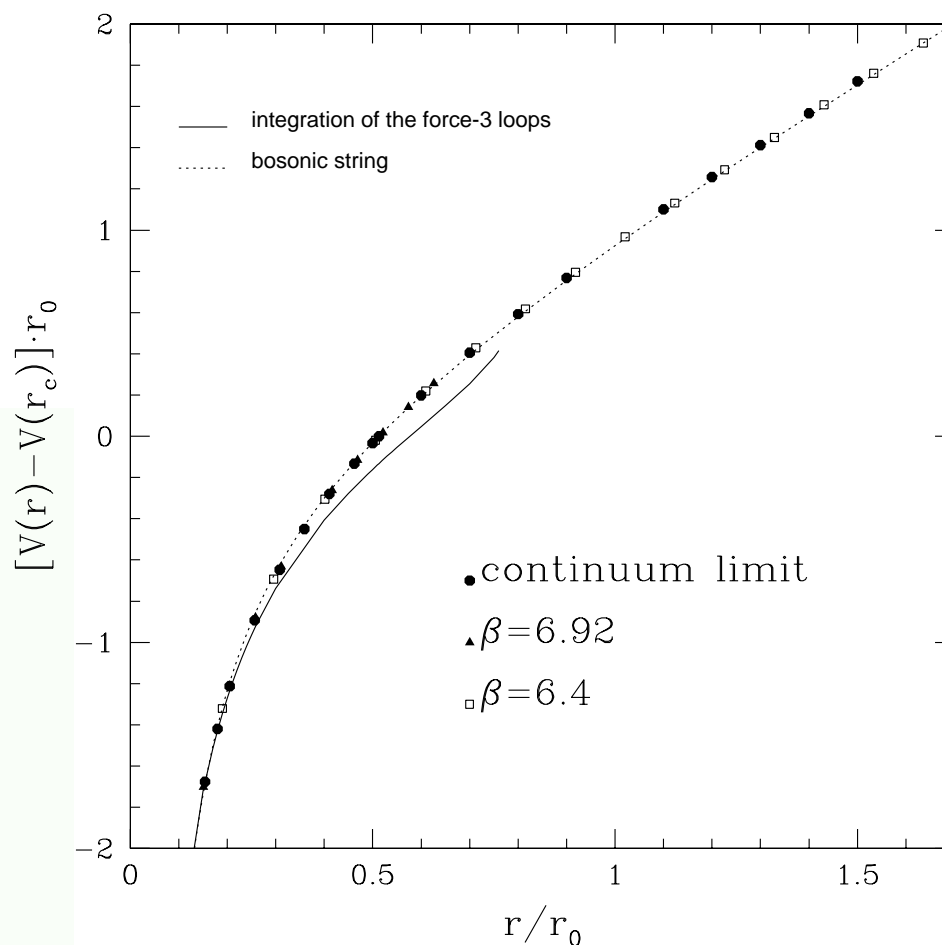
Confinement

● Infinitely Heavy Quarks ... Picture in Quantum Mechanics

$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

$$\sigma \sim 470 \text{ MeV}$$

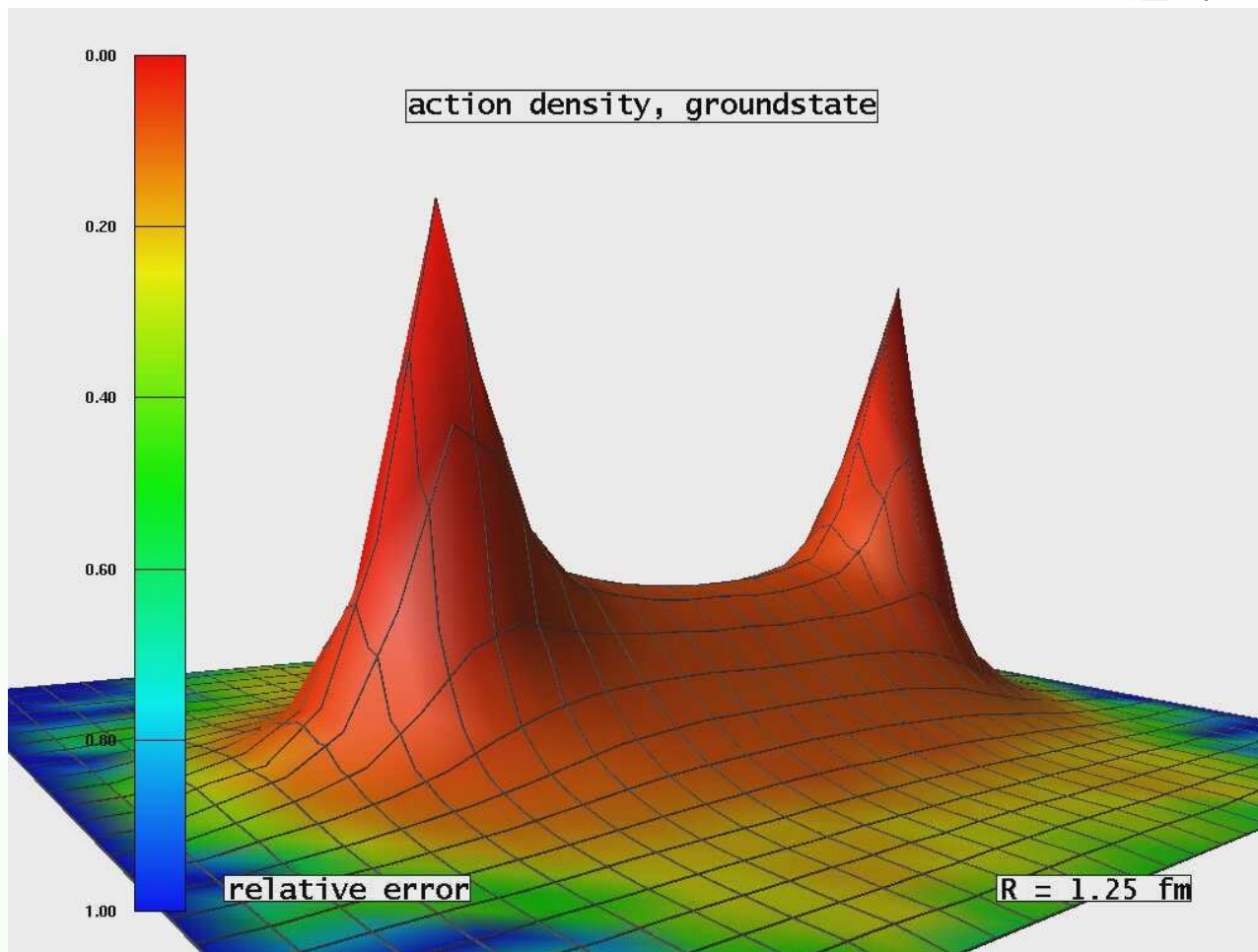
Necco & Sommer
he-lq/0108008



Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force = $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$

Bali, *et al.*
he-lq/0512018



Confinement

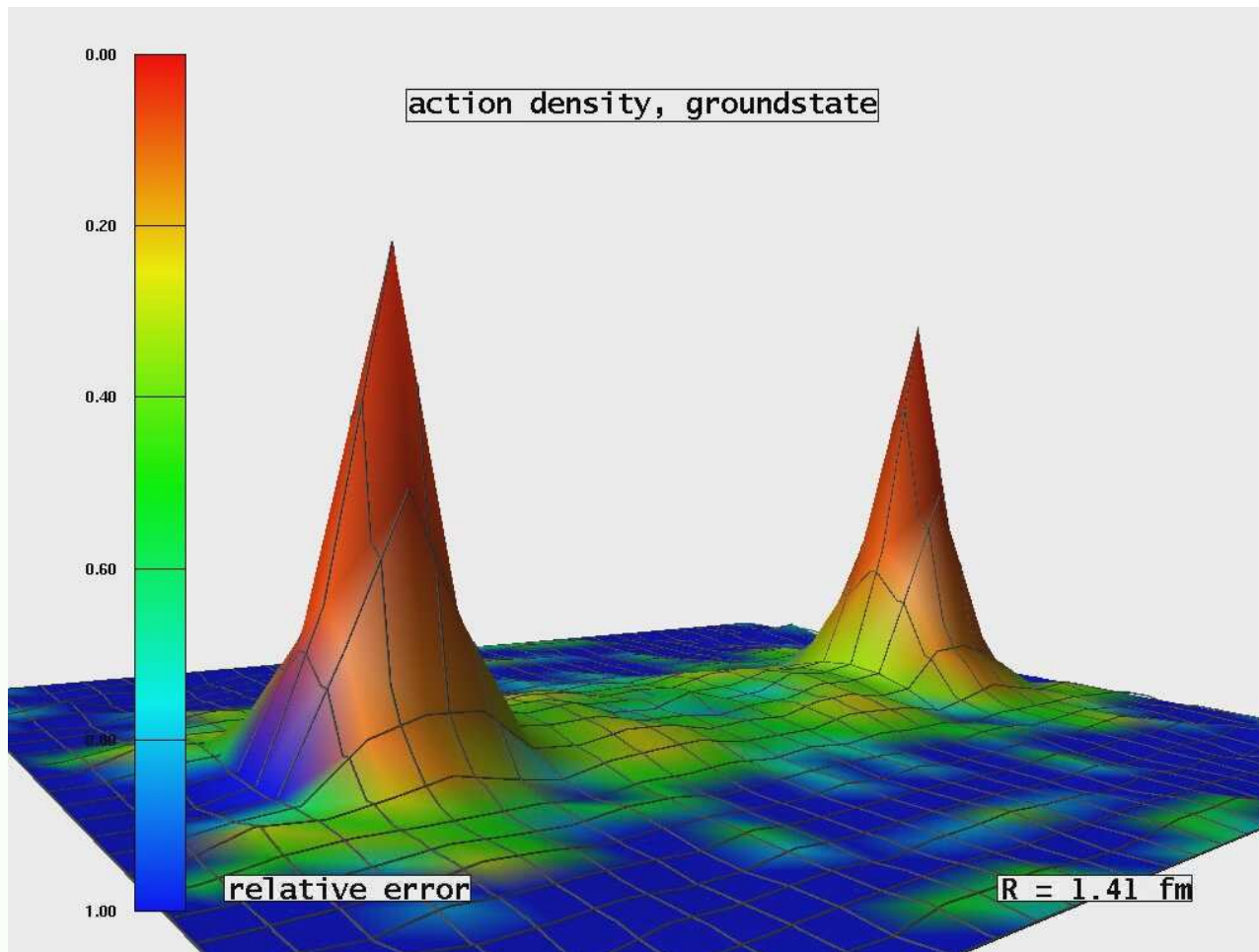
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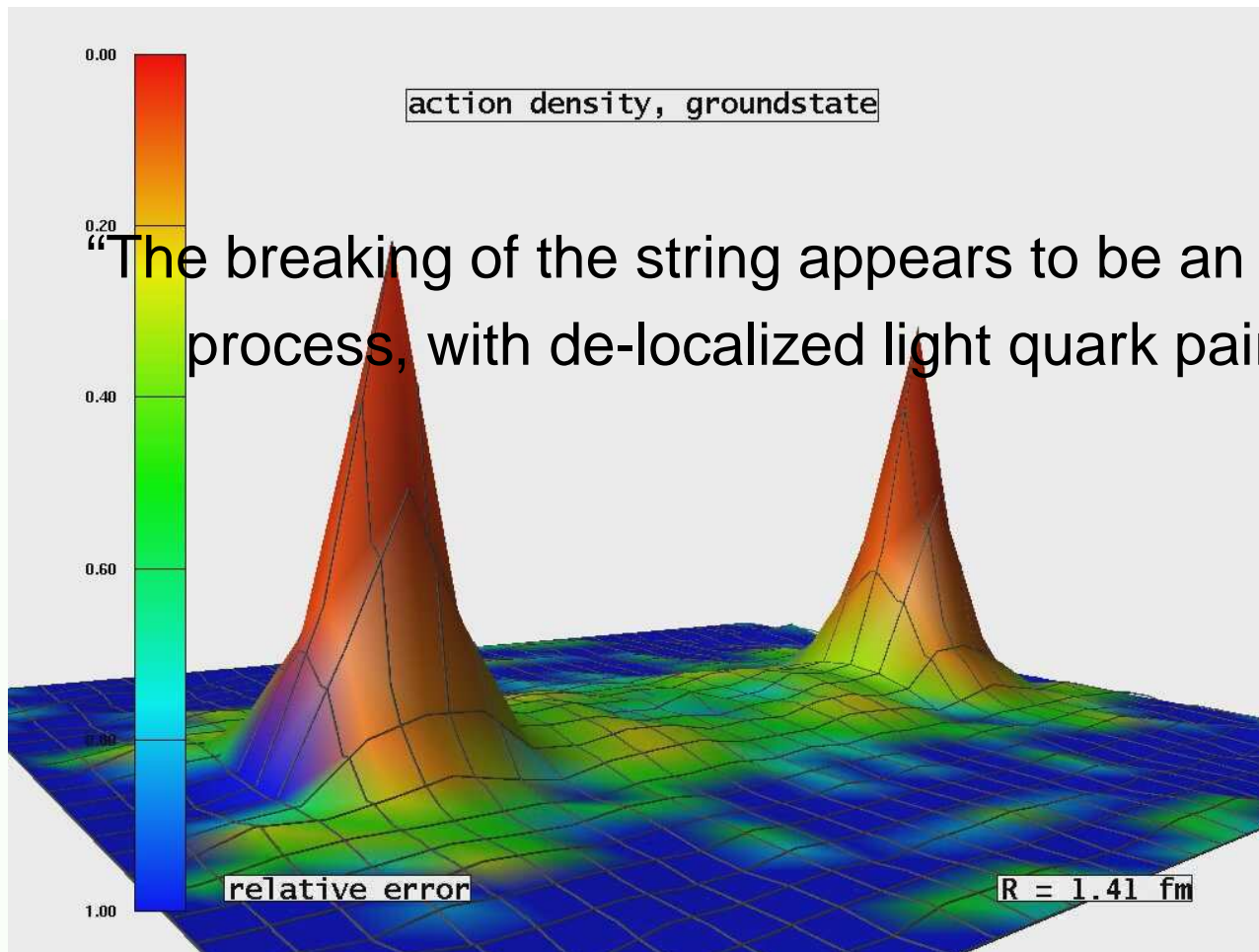
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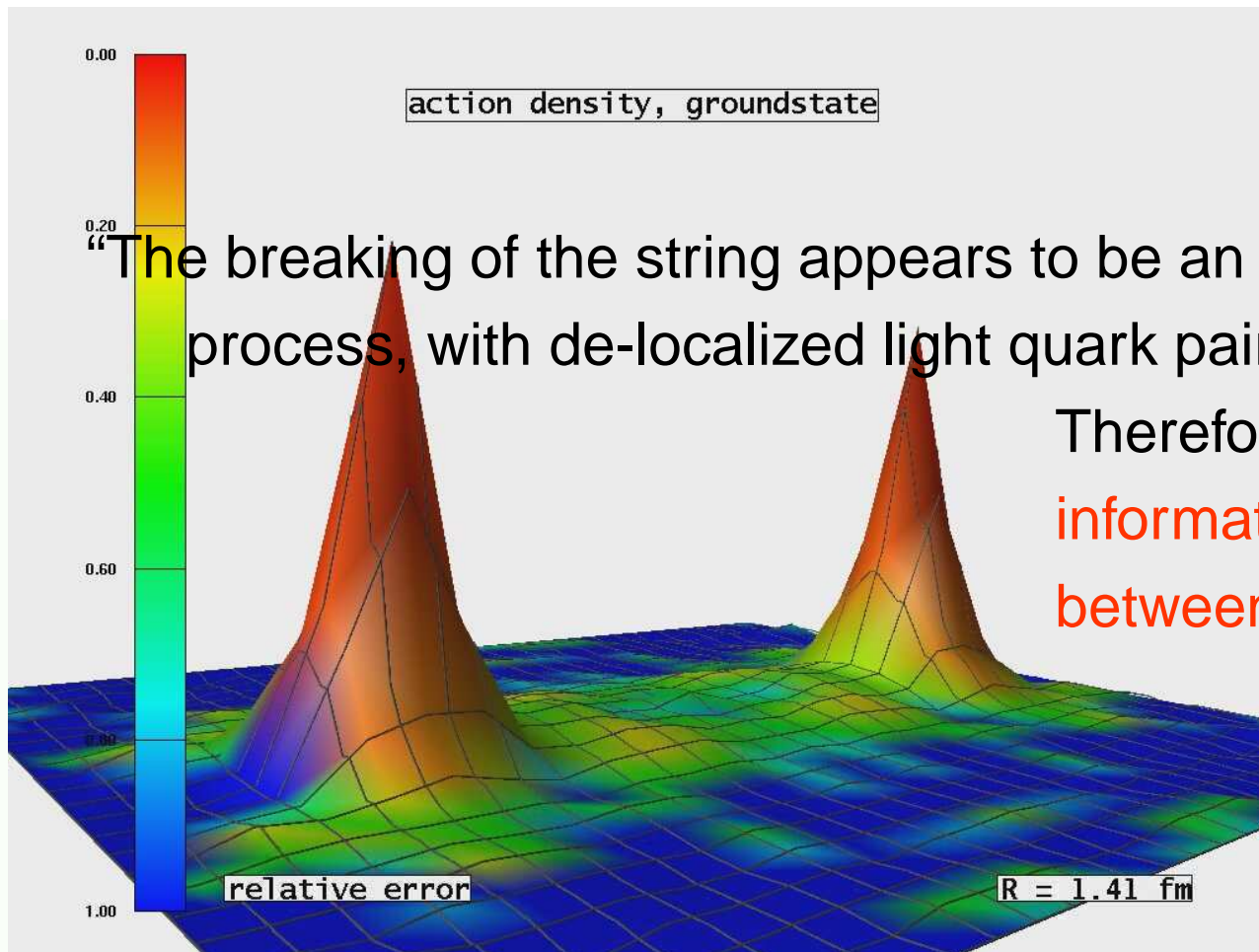
“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”



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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Therefore ... No
information on *potential*
between light-quarks.



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons



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ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
– Goldstone’s Theorem –
impacts upon every pseudoscalar meson



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032



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Diehl & Hiller
he-ph/0105194



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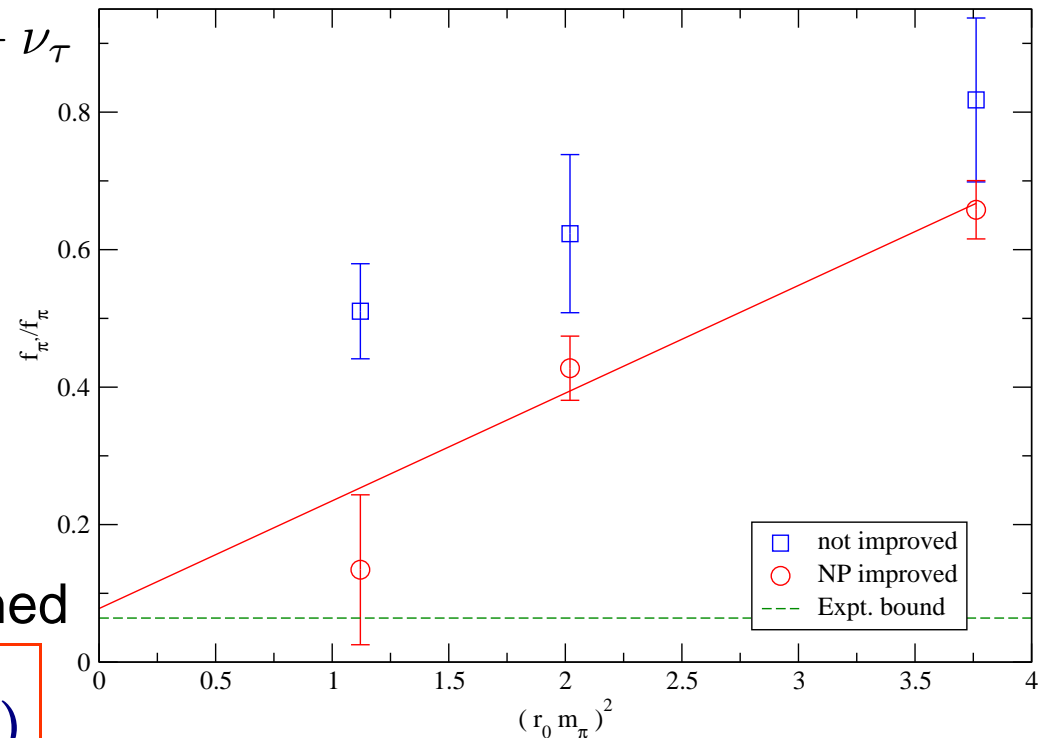
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- Lattice-QCD check:
 $16^3 \times 32$,
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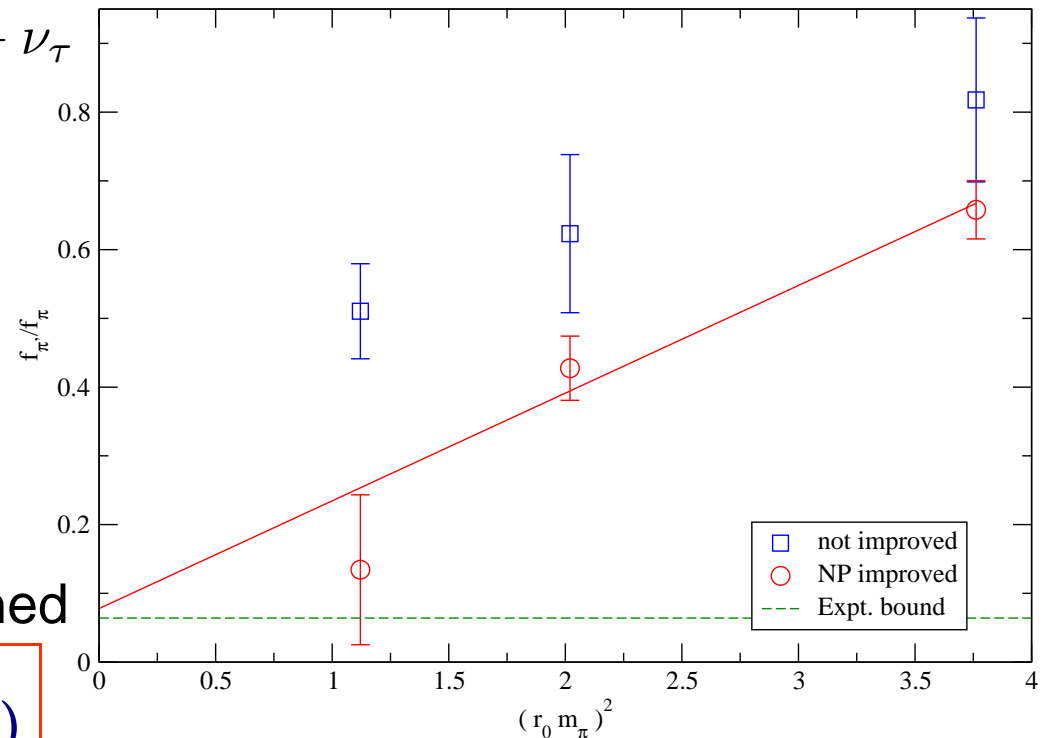
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



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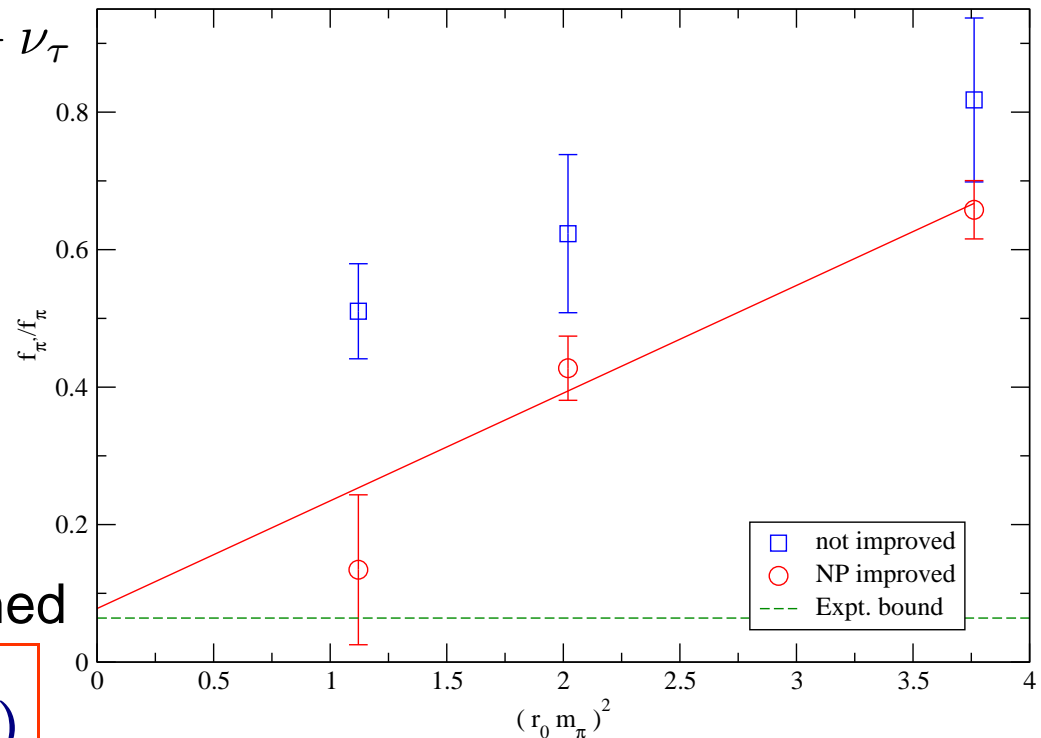
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- The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.



DSE-based Faddeev Equation





Cloët *et al.*

- arXiv:0710.2059 [nucl-th]
- arXiv:0710.5746 [nucl-th]
- arXiv:0804.3118 [nucl-th]

– arXiv:0812.0416 [nucl-th] – *Survey of nucleon EM form factors*

DSE-based Faddeev Equation



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 $M_N = 1.18, M_\Delta = 1.33$
 - allow for pseudoscalar meson contributions



Faddeev Equation

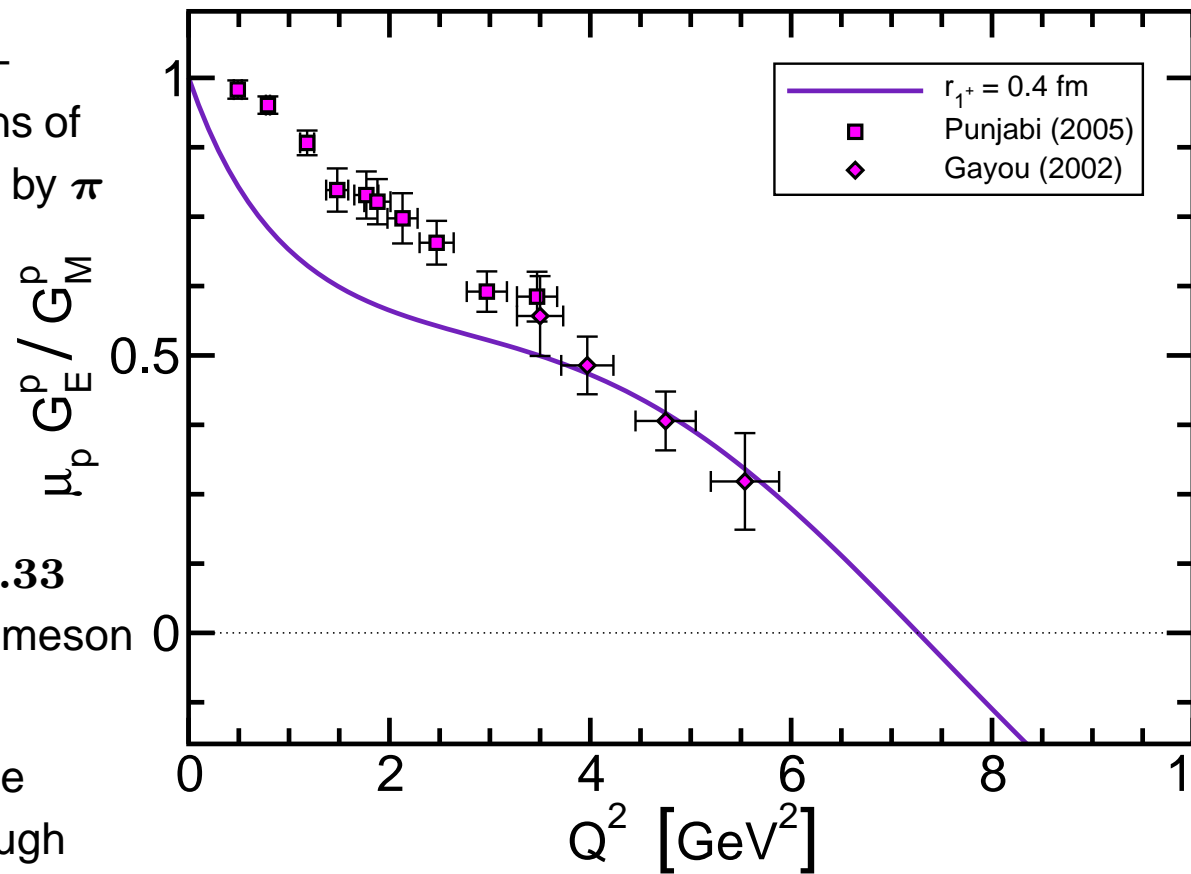
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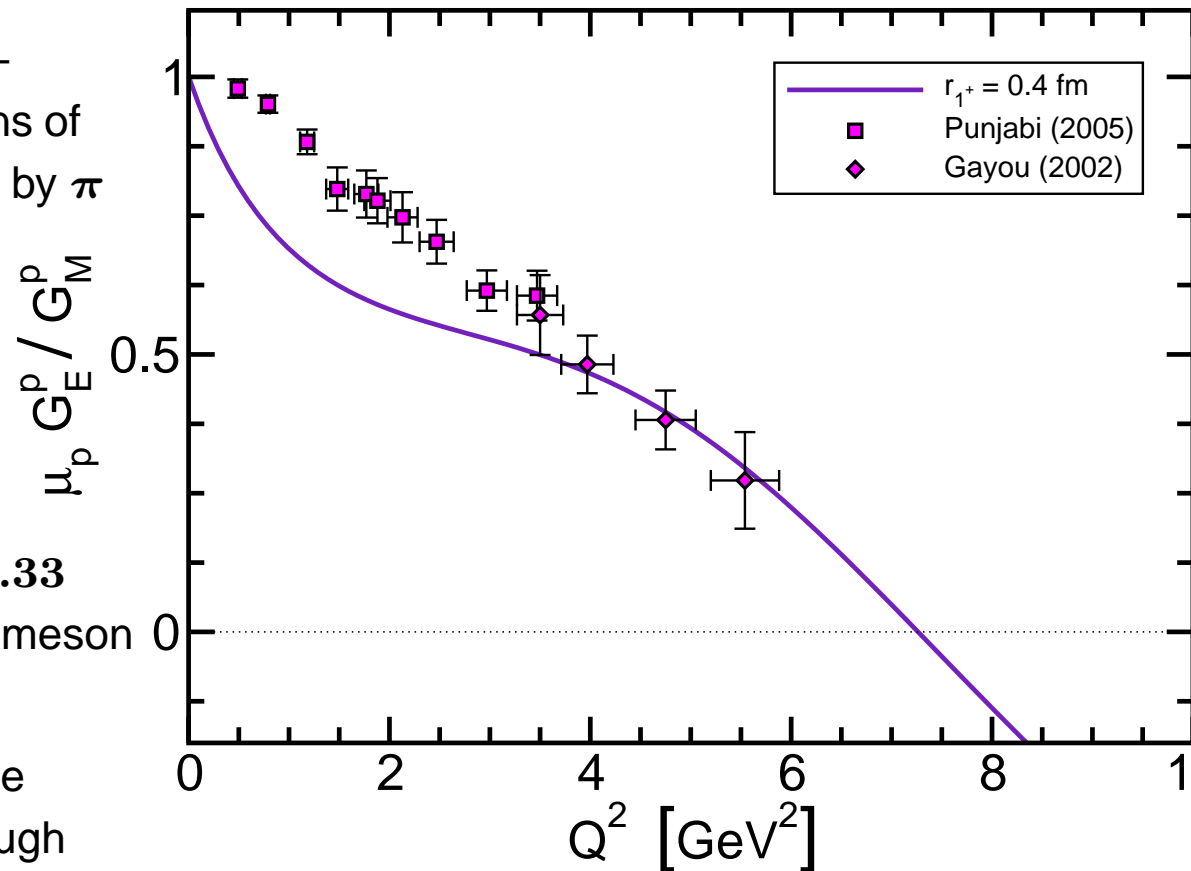
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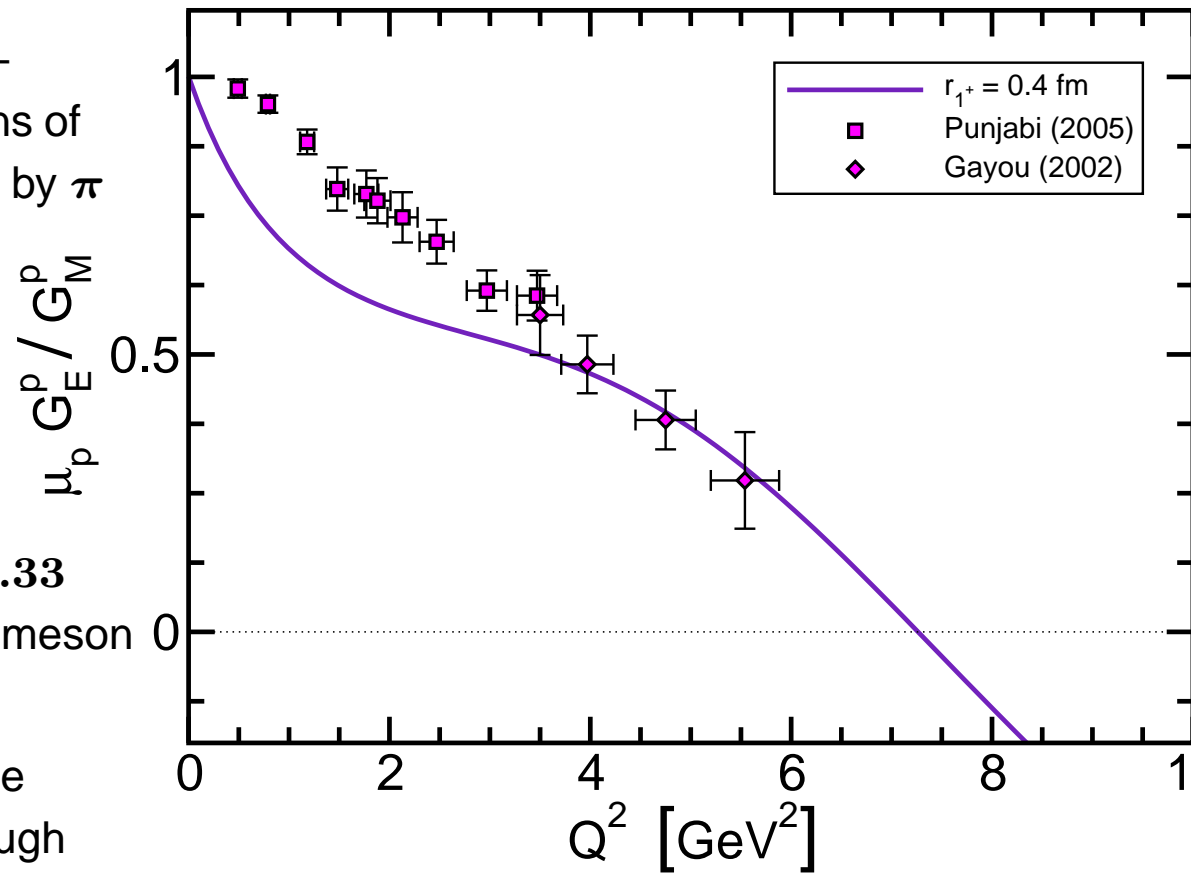
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- Always a zero but position depends on details of current



Ratio of Neutron Pauli & Dirac Form Factors

$$\frac{\hat{Q}^2}{(\ln \hat{Q}^2 / \hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

$$\hat{\Lambda} = \Lambda / M_N = 0.44$$

Ensures proton ratio
constant for $\hat{Q}^2 \geq 4$

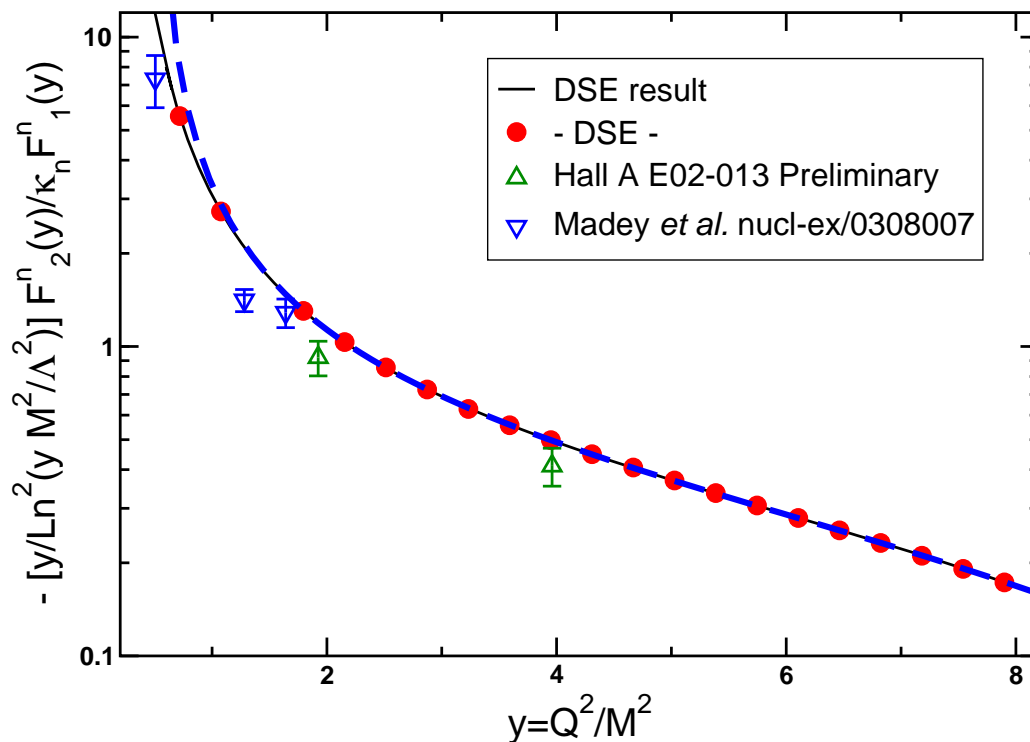


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Pion Cloud

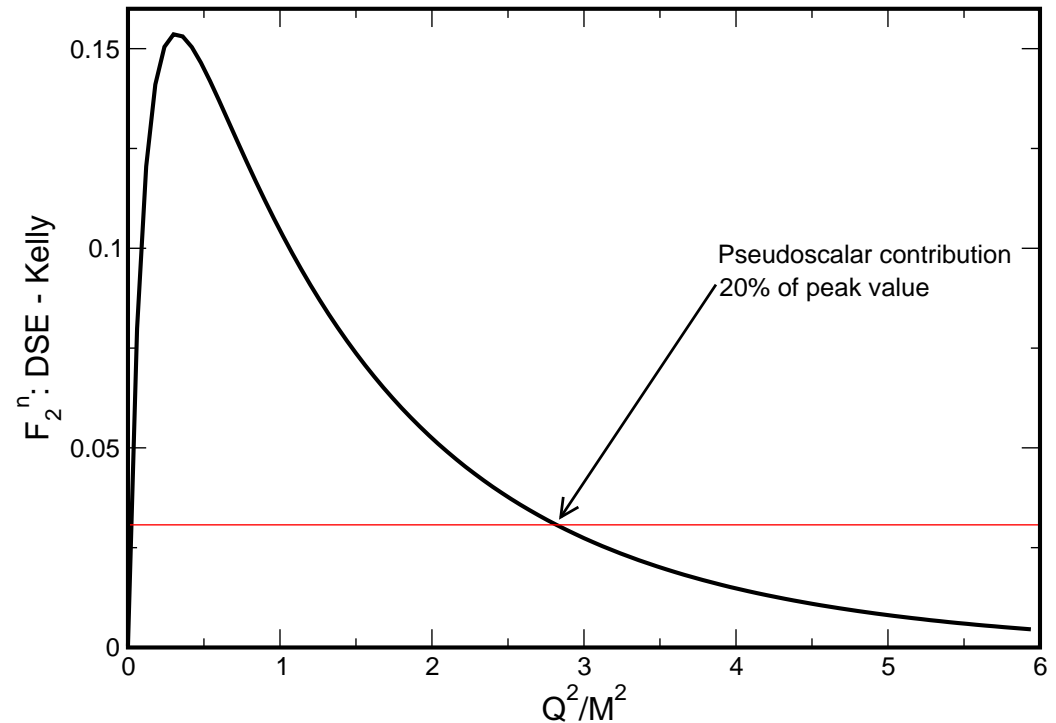
F2 – neutron

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Pion Cloud

F2 – neutron

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Pion Cloud

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- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4 M_N^2$

